

Engineering and Physical Sciences Research Council Doctoral Landscape Award

PROJECT TITLE: The Khintchine-Groshev Theorem: Beyond Removing Monotonicity

Lead Supervisor Details:

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Project details: This project is primarily in the field of Diophantine Approximation, which studies how well real numbers can be approximated by rational numbers and variations on this theme. Sets encountered in Diophantine Approximation are often fractal-like and it is natural to use notions from Fractal Geometry to study them. Diophantine Approximation and Fractal Geometry have found wide-ranging applications in areas such as medicine, materials science, geology, and wireless communications, and have attracted attention of several Fields Medallists.

A classical theorem of Khintchine (1924) states that the set of real numbers which can be approximated at a given rate by rationals has zero or full Lebesgue measure depending on whether a certain sum converges/diverges. This result relies on a monotonicity condition. Duffin and Schaeffer (1941) showed that this monotonicity assumption cannot be removed and conjectured the correct statement without monotonicity. Resolving the Duffin-Schaeffer Conjecture was one of the biggest unsolved problems in Diophantine Approximation until its resolution by Koukoulopoulos and 2022 Fields Medallist Maynard (2020). The Khintchine-Groshev Theorem generalises Khintchine's Theorem to higher dimensions and originally required monotonicity conditions. Beresnevich and Velani (2010) showed that monotonicity could be dropped from the homogeneous Khintchine-Groshev Theorem except when the result reduced to Khintchine's Theorem.

In a series of recent works ([Allen+Ramirez, 2023], [Hauke, 2025], [Kim, 2025], [Kim, 2026+], [Allen+Ramirez+Hauke-Treuer, 2026+]) it has been shown that in all except the classical one-dimensional case (where monotonicity is known to be absolutely necessary), we can also remove monotonicity from the inhomogeneous Khintchine-Groshev Theorem. The aim of this project will be to investigate questions related to the Khintchine-Groshev Theorem beyond that of removing monotonicity.

Possible directions for this project include:

1. Applications of the "Independence Inheritance" phenomenon. In [Allen+Ramirez, 2023], we established an independence inheritance phenomenon, showing that independence of certain sets in dimension n implies k -fold more independence of related sets in $n+k$ dimensions. Our initial motivation for proving this independence inheritance phenomenon was studying the question of whether we could remove monotonicity from the inhomogeneous Khintchine-Groshev Theorem. However, this result is very general and one could investigate further applications of independence inheritance, both in Diophantine Approximation and beyond.

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2. Khintchine-Groshev Theorem on Fractals. Recently, there has been groundbreaking work of Khalil and Luethi (2023), and Bénard, He, and Zhang (2025 and 2026+) proving analogues of Khintchine's classical theorem in self-similar sets (a very natural class of fractals). It would be natural to investigate whether one can prove an analogue of the Khintchine-Groshev Theorem in Fractals.

3. Multivariable Khintchine-Groshev Theorem and Matrix Approximation. In recent work, Zhang, Wang, and Li (2023) studied a problem on "Inhomogeneous multiplicative Diophantine Approximation on Matrix Approximation". To prove their results, they use a mass transference principle for systems of linear forms [Allen+Beresnevich, 2018] and the Khintchine-Groshev Theorem. The aim would be to develop these results further by relaxing the monotonicity condition Zhang, Wang, and Li assume and studying a question they pose about extending their result from dealing with single variable approximation functions to multi-variable ones.

Project specific entry requirements: A degree in Mathematics or at least with a significant mathematical component. A final/expected degree classification of 2:1 or above (or relevant equivalent grade) would be preferable.

Potential PhD programme of study: PhD in Mathematics

Department: Mathematics and Statistics

Location: Harrison/Laver, Streatham

Please direct project specific enquiries to: Demi Allen (d.d.allen@exeter.ac.uk)

Please ensure you read the entry requirements for the potential programme you are applying for.

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