Degrees of Freedom of $M \times N$ SISO X Channel with Synergistic Alternating CSIT

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Abstract: **In this paper, we investigate degrees of freedom (DoF) of the $M \times N$ single input single output (SISO) X channel with alternating channel state information at the transmitter (CSIT). It is known that sum DoF of 2-user SISO X channel with synergistic alternating CSIT is the same as the sum DoF of 2-user ($M = N = 2$) SISO X channel with perfect CSIT [8]. As an extension of the 2-user case, achievable schemes of the $M \times N$ SISO X channel with synergistic alternating CSIT are proposed. It is shown that the proposed $M \times N$ X channel schemes with synergistic alternating CSIT achieve the sum DoF $2M/(M+1)$, which is better than the weighted sum DoF of the conventional schemes for $M \leq N$. The DoF of the proposed scheme with $M = N = K$ is strictly better than the best known DoF for the K-user X channel with delayed CSIT.**

Index Terms: **Channel state information at the transmitter (CSIT), degrees of freedom (DoF), interference alignment, single input single output (SISO) X channel**

I. INTRODUCTION

INTERFERENCE alignment (IA) is an important technique to manage interference in the wireless communication networks. Most of the previous works on the interference alignment [1], [2] assume that perfect channel state information (CSI) is available at all transmitters. However, in practical communication environment, it is very difficult for transmitters to use the perfect CSI because CSI should be instantaneously available at the transmitter without errors. Thus, it is desirable to minimize the required channel state information at the transmitter (CSIT). Maddah-Ali and Tse [3] proved that completely outdated CSIT can still be useful even if the channel states are completely independent. In the multi-user wireless communication systems, some of channels are quasi-static, that is, CSI of those channels does not change over several time slots. Then the delayed CSIT can be considered as an instantaneous perfect CSIT in the next time slots. Thus, both the delayed and the instantaneous perfect CSITs can be assumed to coexist in the real multi-user wireless communication systems. Tandon et al. [7] suggested an alternating CSIT model in the broadcasting channel, where the availability of the delayed and the perfect CSITs varies over time. Recently, Wagdyy et al. [8], [9] introduced new achievable

schemes for 2-user single input single output (SISO) X channel with synergistic alternating CSIT. These schemes achieve 4/3 sum degrees of freedom (DoF), which is the theoretical upper bound for 2-user SISO X channel with the perfect CSIT.

In this paper, based on the results in [8], [9], an achievable scheme is proposed for the $M \times N$ SISO X channel with synergistic alternating CSIT. Also, sum DoF for the proposed $M \times N$ SISO X channel scheme with synergistic alternating CSIT is derived, which is better than the weighted sum DoF of the conventional schemes for $M \leq N$. The DoF of the proposed scheme with $M = N = K$ is strictly better than the best known DoF for the K-user X channel with delayed CSIT. The achievable scheme was proposed for the K-user SISO X channel with synergistic alternating CSIT in [10]. However, the paper [10] is simultaneously studied at the time of the paper [11] submitted by the author of this paper to the arXiv. In addition, there are differences between the achievable schemes in [10] and our proposed achievable schemes. We generalize the number of transceivers to $M \times N$ and proposed four achievable schemes that can be applied according to M and N conditions. Also, we compare the DoF of the proposed scheme and the DoF of the conventional scheme for all $M \times N$.

The rest of this paper is organized as follows: In Section II, the system model of $M \times N$ SISO X channel is described. Then, an achievable scheme for $M \times N$ SISO X channel with synergistic alternating CSIT is proposed and its sum DoF is drived in Section III. In Section IV, we compare the DoF of the proposed scheme with the weighted sum DoF of the conventional scheme. Finally, conclusion is given in Section V.

II. SYSTEM MODEL

We consider $M \times N$ SISO X channel, where each transmitter T_j, $j = 1, \cdots, M$, transmits independent message W_{ij} to each receiver R_i, $i = 1, \cdots, N$. Let $X_j(t) = f_{1j}(t)W_{1j} + f_{2j}(t)W_{2j} + \cdots + f_{Nj}(t)W_{Nj}$ be the signal transmitted from the jth transmitter T_j at the time slot t, where $f_{ij}(t)$ is the precoding coefficient for the message W_{ij}. The received signal at the ith receiver at the time slot t is given as

$$Y_i(t) = \sum_{j=1}^{M} h_{ij}(t) X_j(t) + N_i(t), \tag{1}$$

where $h_{ij}(t)$ denotes the channel coefficient between the jth transmitter and the ith receiver and $N_i(t) \sim \mathcal{CN}(\prime, \infty)$ is the circularly symmetric white Gaussian noise with zero mean and unit variance at the receiver R_i. The power constraint is assumed to be $E||X(t)||^2 \leq P$. Let $r_{ij}(P) = \log_2 |W_{ij}|/n$ denote the achievable rate per channel use of W_{ij} for the transmission power P, where $|W_{ij}|$ denotes the alphabet size of W_{ij}.
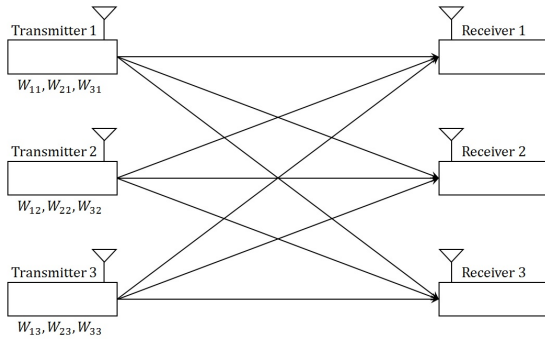
Fig. 1.   3-user SISO X channel.

The DoF region $\mathcal{D}$ of the $M \times N$ X channel is defined as the set of all real non-negative tuples $(d_{11}, d_{12}, \cdots, d_{MN}) \in R_+^{MN}$, where $d_{ij} = \lim_{P \to \infty} r_{ij}(P)/\log P$. The sum DoF of $M \times N$ X channel is defined as [1]

$$\text{DoF} = \max_{(d_{11}, d_{12}, \cdots, d_{MN}) \in \mathcal{D}} \sum_{i=1}^{M} \sum_{j=1}^{N} d_{ij}. \qquad (2)$$

It is considered that there are three different states of availability of CSIT for the receiver side as [8]:

• Perfect CSIT (P state): CSIT is available at the transmitters instantaneously and without error.
• Delayed CSIT (D state): CSIT is available at the transmitters with some delay and without error.
• No CSIT (N state): CSIT is not available at the transmitters at all.

If the receiver R_i is in the P state, then each transmitter knows the perfect CSI $h_{ij}(t)$, $1 \le j \le M$ at time slot t. In the same manner, if the receiver R_i is in the D state, then each transmitter knows the delayed CSI $h_{ij}(t)$, $1 \le j \le M$ at time slot $t + \tau$ with the delay τ.

III. PROPOSED ACHIEVABLE SCHEME

In this section, we propose achievable schemes for the 3-user SISO X channel ($M = N = 3$) and extend it to the general $M \times N$ SISO X channel.

A. Achievable Scheme for the 3-user SISO X Channel

First, we extend the achievable schemes in [8] to the 3-user SISO X channel in Fig. 1. Using synergistic alternating CSIT [8], $4/3$ DoF for the 2-user SISO X channel is achieved. In the proposed scheme for the 3-user case, each transmitter transmits independent messages to three receivers over six time slots. Similar to [8], the proposed achievable scheme for the 3-user case is composed of two separate phases described as below.

Phase 1) In this phase, each transmitter transmits its messages during three time slots. At time slot $t = i$, $1 \le i \le 3$, all three transmitters transmit their messages for the receiver R_i, i.e., $X_j(t = i) = W_{ij}$, $1 \le i, j \le 3$. Then, the transmitted

signals at time slot t, $t = 1, 2, 3$, are given as

$$\begin{aligned} X_1(1) &= W_{11}, \ X_2(1) = W_{12}, \ X_3(1) = W_{13} \\ X_1(2) &= W_{21}, \ X_2(2) = W_{22}, \ X_3(2) = W_{23} \\ X_1(3) &= W_{31}, \ X_2(3) = W_{32}, \ X_3(3) = W_{33}. \end{aligned} \qquad (3)$$

Also, the received signals $Y_i(t)$, $1 \le i, t \le 3$, at the receiver R_i are given as

$$\begin{aligned} Y_1(1) &= h_{11}(1)W_{11} + h_{12}(1)W_{12} + h_{13}(1)W_{13} \\ &\equiv L_1^1(W_{11}, W_{12}, W_{13}) \\ Y_2(1) &= h_{21}(1)W_{11} + h_{22}(1)W_{12} + h_{23}(1)W_{13} \\ &\equiv I_2^1(W_{11}, W_{12}, W_{13}) \\ Y_3(1) &= h_{31}(1)W_{11} + h_{32}(1)W_{12} + h_{33}(1)W_{13} \\ &\equiv I_3^1(W_{11}, W_{12}, W_{13}), \end{aligned} \qquad (4)$$

$$\begin{aligned} Y_1(2) &= h_{11}(2)W_{21} + h_{12}(2)W_{22} + h_{13}(2)W_{23} \\ &\equiv I_1^1(W_{21}, W_{22}, W_{23}) \\ Y_2(2) &= h_{21}(2)W_{21} + h_{22}(2)W_{22} + h_{23}(2)W_{23} \\ &\equiv L_2^1(W_{21}, W_{22}, W_{23}) \\ Y_3(2) &= h_{31}(2)W_{21} + h_{32}(2)W_{22} + h_{33}(2)W_{23} \\ &\equiv I_3^2(W_{21}, W_{22}, W_{23}), \end{aligned} \qquad (5)$$

$$\begin{aligned} Y_1(3) &= h_{11}(3)W_{31} + h_{12}(3)W_{32} + h_{13}(3)W_{33} \\ &\equiv I_1^2(W_{31}, W_{32}, W_{33}) \\ Y_2(3) &= h_{21}(3)W_{31} + h_{22}(3)W_{32} + h_{23}(3)W_{33} \\ &\equiv I_2^2(W_{31}, W_{32}, W_{33}) \\ Y_3(3) &= h_{31}(3)W_{31} + h_{32}(3)W_{32} + h_{33}(3)W_{33} \\ &\equiv L_3^1(W_{31}, W_{32}, W_{33}), \end{aligned} \qquad (6)$$

where L_i^k denotes the kth linear combination of three desired messages (W_{i1}, W_{i2}, W_{i3}) for the receiver R_i and I_i^k denotes the kth interference signal to the receiver R_i, which is composed of three interference messages.

Phase 2) In this phase, all three transmitters transmit their messages for a pair of receivers at each time slot. Since there are total three receiver pairs, three time slots are needed in this phase. Note that the transmitted signal at the transmitter j is precoded for interference cancellation by using the messages $\{W_{1j}, W_{2j}, W_{3j}\}$ and the delayed CSITs in the phase 1 and the perfect CSITs. That is, at time slot $t = 4$, R_1 and R_2 are P state, i.e., T_j knows $h_{1j}(4)$ and $h_{2j}(4)$. Also, R_1 is D state at time slot $t = 2$ and R_2 is D state at time slot $t = 1$, i.e., T_j knows $h_{1j}(2)$ and $h_{2j}(1)$. By using these CSITs, each transmitter transmits the precoded signal at time slot $t = 4$ as follows;

$$\begin{aligned} X_1(4) &= h_{21}^{-1}(4)h_{21}(1)W_{11} + h_{11}^{-1}(4)h_{11}(2)W_{21} \\ X_2(4) &= h_{22}^{-1}(4)h_{22}(1)W_{12} + h_{12}^{-1}(4)h_{12}(2)W_{22} \\ X_3(4) &= h_{23}^{-1}(4)h_{23}(1)W_{13} + h_{13}^{-1}(4)h_{13}(2)W_{23}. \end{aligned} \qquad (7)$$

Then, the received signals at time slot $t = 4$ are given as

$$
\begin{aligned}
Y_1(4) &= h_{11}(4)h_{21}^{-1}(4)h_{21}(1)W_{11} + h_{12}(4)h_{22}^{-1}(4)h_{22}(1)W_{12} \\
&\quad + h_{13}(4)h_{23}^{-1}(4)h_{23}(1)W_{13} + h_{11}(2)W_{21} + h_{12}(2)W_{22} \\
&\quad + h_{13}(2)W_{23} \\
&\equiv L_1^2(W_{11}, W_{12}, W_{13}) + I_1^1(W_{21}, W_{22}, W_{23}) \\
Y_2(4) &= h_{21}(4)h_{11}^{-1}(4)h_{11}(2)W_{21} + h_{22}(4)h_{12}^{-1}(4)h_{12}(2)W_{22} \\
&\quad + h_{23}(4)h_{13}^{-1}(4)h_{13}(2)W_{23} + h_{21}(1)W_{11} + h_{22}(1)W_{12} \\
&\quad + h_{23}(1)W_{13} \\
&\equiv L_2^2(W_{21}, W_{22}, W_{23}) + I_2^1(W_{11}, W_{12}, W_{13}) \\
Y_3(4) &= h_{31}(4)h_{21}^{-1}(4)h_{21}(1)W_{11} + h_{32}(4)h_{22}^{-1}(4)h_{22}(1)W_{12} \\
&\quad + h_{33}(4)h_{23}^{-1}(4)h_{23}(1)W_{13} + h_{31}(4)h_{11}^{-1}(4)h_{11}(2)W_{21} \\
&\quad + h_{32}(4)h_{12}^{-1}(4)h_{12}(2)W_{22} + h_{33}(4)h_{13}^{-1}(4)h_{13}(2)W_{23}.
\end{aligned}
\tag{8}
$$

The receiver R_1 receives the second linear combination $L_1^2(W_{11}, W_{12}, W_{13})$ of its desired messages $\{W_{11}, W_{12}, W_{13}\}$ and the interference $I_1^1(W_{21}, W_{22}, W_{23})$. However, R_1 already received the same interference at time slot $t = 2$ such that $Y_1(2) = I_1^1(W_{21}, W_{22}, W_{23})$. Thus, $L_1^2(W_{11}, W_{12}, W_{13})$ can be obtained by subtracting $Y_1(2)$ from $Y_1(4)$. Similarly, the receiver R_2 can also obtain $L_2^2(W_{21}, W_{22}, W_{23})$ by subtracting $Y_2(1)$ from $Y_2(4)$. In fact, the received signal $Y_3(4)$ is not used in our proposed scheme.

At time slot $t = 5$, R_1 and R_3 are P state, i.e., T_j knows $h_{1j}(5)$ and $h_{3j}(5)$. Also, R_1 is D state at time slot $t = 3$ and R_3 is D state at time slot $t = 1$, i.e., T_j knows $h_{1j}(3)$ and $h_{3j}(1)$. By using these CSITs, each transmitter transmits the precoded signal at time slot $t = 5$ as follows;

$$
\begin{aligned}
X_1(5) &= h_{31}^{-1}(5)h_{31}(1)W_{11} + h_{11}^{-1}(5)h_{11}(3)W_{31} \\
X_2(5) &= h_{32}^{-1}(5)h_{32}(1)W_{12} + h_{12}^{-1}(5)h_{12}(3)W_{32} \\
X_3(5) &= h_{33}^{-1}(5)h_{33}(1)W_{13} + h_{13}^{-1}(5)h_{13}(3)W_{33}.
\end{aligned}
\tag{9}
$$

At time slot $t = 6$, R_2 and R_3 are P state, i.e., T_j knows $h_{2j}(6)$ and $h_{3j}(6)$. Also, R_2 is D state at time slot $t = 3$ and R_3 is D state at time slot $t = 2$, i.e., T_j knows $h_{2j}(3)$ and $h_{3j}(2)$. By using these CSITs, each transmitter transmits the precoded signal at time slot $t = 6$ as follows;

$$
\begin{aligned}
X_1(6) &= h_{31}^{-1}(6)h_{31}(2)W_{21} + h_{21}^{-1}(6)h_{21}(3)W_{31} \\
X_2(6) &= h_{32}^{-1}(6)h_{32}(2)W_{22} + h_{22}^{-1}(6)h_{22}(3)W_{32} \\
X_3(6) &= h_{33}^{-1}(6)h_{33}(2)W_{23} + h_{23}^{-1}(6)h_{23}(3)W_{33}.
\end{aligned}
\tag{10}
$$

Then, the received signals at time slot $t = 5, 6$ are also given

Table 1. Synergistic alternating CSIT states for 3-user X channel.

	Phase 1			Phase 2		
Time	1	2	3	4	5	6
R_1	N	D	D	P	P	N
R_2	D	N	D	P	N	P
R_3	D	D	N	N	P	P

as

$$
\begin{aligned}
Y_1(5) &= h_{11}(5)h_{31}^{-1}(5)h_{31}(1)W_{11} + h_{12}(5)h_{32}^{-1}(5)h_{32}(1)W_{12} \\
&\quad + h_{13}(5)h_{33}^{-1}(5)h_{33}(1)W_{13} + h_{11}(3)W_{31} + h_{12}(3)W_{32} \\
&\quad + h_{13}(3)W_{33} \\
&\equiv L_1^3(W_{11}, W_{12}, W_{13}) + I_1^2(W_{31}, W_{32}, W_{33})
\end{aligned}
$$

$Y_2(5)$; this received signal is not used.

$$
\begin{aligned}
Y_3(5) &= h_{31}(5)h_{11}^{-1}(5)h_{11}(3)W_{31} + h_{32}(5)h_{12}^{-1}(5)h_{12}(3)W_{32} \\
&\quad + h_{33}(5)h_{13}^{-1}(5)h_{13}(3)W_{33} + h_{31}(1)W_{11} + h_{32}(1)W_{12} \\
&\quad + h_{33}(1)W_{13} \\
&\equiv L_3^2(W_{31}, W_{32}, W_{33}) + I_3^1(W_{11}, W_{12}, W_{13}),
\end{aligned}
\tag{11}
$$

$Y_1(6)$; this received signal is not used.

$$
\begin{aligned}
Y_2(6) &= h_{21}(6)h_{31}^{-1}(6)h_{31}(2)W_{21} + h_{22}(6)h_{32}^{-1}(6)h_{32}(2)W_{22} \\
&\quad + h_{23}(6)h_{33}^{-1}(6)h_{33}(2)W_{23} + h_{21}(3)W_{31} + h_{22}(3)W_{32} \\
&\quad + h_{23}(3)W_{33} \\
&\equiv L_2^3(W_{21}, W_{22}, W_{23}) + I_2^2(W_{31}, W_{32}, W_{33}) \\
Y_3(6) &= h_{31}(6)h_{21}^{-1}(6)h_{21}(3)W_{31} + h_{32}(6)h_{22}^{-1}(6)h_{22}(3)W_{32} \\
&\quad + h_{33}(6)h_{23}^{-1}(6)h_{23}(3)W_{33} + h_{31}(2)W_{21} + h_{32}(2)W_{22} \\
&\quad + h_{33}(2)W_{23} \\
&\equiv L_3^3(W_{31}, W_{32}, W_{33}) + I_3^2(W_{21}, W_{22}, W_{23}).
\end{aligned}
\tag{12}
$$

At time slot $t = 5, 6$, the receiver R_1 obtains the third linear combination $L_1^3(W_{11}, W_{12}, W_{13})$ by subtracting $Y_1(2)$ from $Y_1(5)$ and the receiver R_2 obtains the third linear combination $L_2^3(W_{21}, W_{22}, W_{23})$ by subtracting $Y_2(3)$ from $Y_2(6)$. Similarly, R_3 obtains $L_3^2(W_{31}, W_{32}, W_{33})$ by subtracting $Y_3(1)$ from $Y_3(5)$ and $L_3^3(W_{31}, W_{32}, W_{33})$ by subtracting $Y_3(2)$ from $Y_3(6)$. Therefore, in the phase 2, each receiver obtains three linear combinations of the desired messages, i.e.,

$$
\begin{aligned}
&R_1; \{L_1^1(W_{11}, W_{12}, W_{13}), L_1^2(W_{11}, W_{12}, W_{13}), L_1^3(W_{11}, W_{12}, W_{13})\} \\
&R_2; \{L_2^1(W_{21}, W_{22}, W_{23}), L_2^2(W_{21}, W_{22}, W_{23}), L_2^3(W_{21}, W_{22}, W_{23})\} \\
&R_3; \{L_3^1(W_{31}, W_{32}, W_{33}), L_3^2(W_{31}, W_{32}, W_{33}), L_3^3(W_{31}, W_{32}, W_{33})\}.
\end{aligned}
$$

As a result, the receiver R_i can decode three desired messages $\{W_{i1}, W_{i2}, W_{i3}\}$ by solving their three linear equations. Thus, sum DoF $9/6 = 3/2$ is achieved for the 3-user SISO X channel by the proposed achievable scheme. As you can see in (7), (9), and (10), each transmitter requires the delayed and the perfect CSITs to precode its transmitting messages.

Table 1 summarizes the required synergistic alternating CSIT states at the receiver side. At time slot 1, R_1 receives the linear combination of the desired messages $\{W_{11}, W_{12}, W_{13}\}$, which correspond to the interference messages to R_2 and R_3. At time slot 4, R_1 receives the second linear combination of the desired messages $\{W_{11}, W_{12}, W_{13}\}$ and the linear combination of the

interference messages by $\{W_{21}, W_{22}, W_{23}\}$. At R_1, by using the interference signal $I_1^1(W_{21}, W_{22}, W_{23})$ received at time slot 2, the interference at time slot 4 is cancelled by precoding the transmitted messages as in (7). That is, the precoding is done at each transmitter by using the current channel coefficients (P CSIT) and the previous channel coefficients (D CSIT). Similarly, the interference of R_1 at time slot 5 can be cancelled by using the interference at time slot 3 and the precoded transmitted signals at time slot 5 in (9). For R_2 and R_3, the interference can be cancelled in the similar manner to obtain three linear combinations of the desired messages. As a result, in the phase 1, if the receiver receives the interference, it is used in the phase 2 as D CSIT. Therefore, the receivers should be D state. Also, if the receiver receives the desired signals, the corresponding CSIT is not used in the phase 2. In the phase 2, if the receiver receives linear combinations of its desired messages and interference, it requires P state for precoder and if the receiver receives only the interference, it is in N state as in Table 1.

In the practical point of view, the variation in the availability of CSIT is practically unavoidable due to the time-varying nature of wireless networks. In phase 1 of the proposed scheme, delayed CSIT and No CSIT are required and thus the CSIT requirements can be mitigated at the initial state of communication. Furthermore, the alternating CSIT can be intentionally designed by slightly converting the given channel states. That is, the form of CSIT states may also be deliberately varied depending on the design of CSIT states.

It is clear that in the proposed achievable scheme, the CSIT states in Table 1 can be columnwisely permuted within each phase. Since each phase uses three time slots, there are $3!3! = 36$ possible CSIT states with rescheduled transmission order. Therefore, Table 1 and its permutations are the minimum requirements of CSIT states for the proposed scheme. Also, the proposed scheme can be applied to the better channel states, i.e., N state can be D or P and D states can be P state. In fact, the proposed achievable schemes are motivated from the alternating CSIT model in [7]. In the time-varying channel, the variation in the availability of CSIT is practically unavoidable. Furthermore, the alternating CSIT in Table 1 can be intentionally designed by slightly converting the given channel states. In Section 4, we will show the benefits of the proposed achievable scheme with the alternating CSIT compared to the conventional scheme.

B. Achievable Schemes for $M \times N$ SISO X Channel

In the $M \times N$ SISO X channel, we have MN independent messages W_{ij}, $i = 1, \cdots, N$, $j = 1, \cdots, M$. Therefore, each receiver requires M linear combinations of M desired messages to recover messages during the given channel uses. Similar to the 3-user case, the proposed achievable schemes for $M \times N$ SISO X channel with synergistic alternating CSIT are composed of two phases. Depending on the number of users, the following four cases of the achievable schemes are considered as below.

i) $M \geq N$ except for even M and odd N:
Phase 1) In this phase, N time slots are used. At time slot i, all transmitters transmit their desired messages to the receiver R_i, i.e., $X_j(i) = W_{ij}$, $1 \leq i \leq N$, $1 \leq j \leq M$.
Phase 2) In this phase, all transmitters transmit linear com-

binations of two desired messages $\{W_{i_1 j}, W_{i_2 j}\}$, $1 \leq j \leq M$, to each pair of two receivers $\{R_{i_1}, R_{i_2}\}$ at each time slot. In other words, at the time slot t, the jth transmitter transmits $X_j(t) = f_{i_1 j}(t)W_{i_1 j} + f_{i_2 j}(t)W_{i_2 j}$, $1 \leq j \leq M$, where $f_{i_1 j}(t)$ and $f_{i_2 j}(t)$ are precoding coefficients. Thus, all transmitters transmit $\binom{N}{2}$ linear combinations of two desired messages to each pair of two receivers over $\binom{N}{2}$ time slots. During these $\binom{N}{2}$ time slots, each receiver receives $N - 1$ linear combinations of its desired messages and interferences from M transmitters among $\binom{N}{2}$ received signals. Since each receiver receives one linear combination of M desired messages from M transmitters in the phase 1, in order to seperate M desired messages, each receiver needs additional $M - 1$ linear combinations of M desired messages. For that purpose, all transmitters transmit repeatedly until all receivers have received $M - 1$ linear combinations of the desired messages in the phase 2. Let α be the repeated number of $\binom{N}{2}$ transmissions in the phase 2. Since each receiver obtains $N - 1$ linear combinations of its desired messages during one repetition, the repeated number α should satisfy $\alpha(N-1) = M-1$. Then, the total number of transmissions in the phase 2 is equal to $(M-1)\binom{N}{2}/(N-1) = N(M-1)/2$. The transmitted messages are precoded for interference cancellation using the previous time slot CSI as in (7), (9), and (10). The transmitted signals are given as in Table 2, where $W_i = \{W_{ij} | j = 1, 2, \cdots, M\}$ is the desired messages to the receiver R_i.

Then, the received signals are given as in Table 3, where '-' means useless received interference signal.

In the phase 1, each receiver obtains one linear combination of the desired messages. In the phase 2, each receiver obtains $M - 1$ additional linear combinations of its desired messages by subtracting the received interference at the phase 1. For example, R_1 obtains $L_1^2(W_1)$ by substracting $I_1^1(W_2)$ at time slot 2 from $L_1^2(W_1) + I_1^1(W_2)$ at time slot $N + 1$. Using $N + N(M - 1)/2$ channel uses, each receiver has M linear combinations of its desired messages, i.e.,

$$R_1; \{L_1^1(W_1), L_1^2(W_1), \cdots, L_1^M(W_1)\}$$
$$R_2; \{L_2^1(W_2), L_2^2(W_2), \cdots, L_2^M(W_2)\}$$
$$\cdots$$
$$R_N; \{L_N^1(W_N), L_N^2(W_N), \cdots, L_N^M(W_N)\}.$$

Therefore, each receiver can decode M desired messages by solving M linear equations and the sum DoF $MN/(N + N(M - 1)/2) = 2M/(M + 1)$ is achieved by the proposed scheme.

Synergistic alternating CSIT states used in the proposed scheme are listed in Table 4. Similar to the 3-user case, in the phase 1, the receiver that receives the interference has D state to use CSIT in the phase 2 and the receiver that receives the desired signals has N state. In the phase 2, the receiver that receives linear combinations of its desired messages and interferences has P state for precoder and the receiver that receives only the interferences has N state as in Table 4.

In the proposed scheme, the CSIT states in Table 4 can be columnwisely permuted among columns within each phase, and thus there are $N!(N(M - 1)/2)!$ possible CSIT states with

Table 2. Transmitted signals for $M \geq N$ except for even M and odd N.

	Phase 1				Phase 2					
Time	1	2	$\cdots$	N	$N+1$	$N+2$	$\cdots$	$N+\binom{N}{2}$	$\cdots$	$N+\dfrac{N(M-1)}{2}$
Tx	W_1	W_2	$\cdots$	W_N	W_1, W_2	W_1, W_3	$\cdots$	W_{N-1}, W_N	$\cdots$	W_{N-1}, W_N

Table 3. Received signals for $M \geq N$ except for even M and odd N.

	Phase 1				Phase 2			
Time	1	2	$\cdots$	N	$N+1$	$N+2$	$\cdots$	$N+\dfrac{N(M-1)}{2}$
R_1	$L_1^1(W_1)$	$I_1^1(W_2)$	$\cdots$	$I_1^{N-1}(W_N)$	$L_1^2(W_1)+I_1^1(W_2)$	$L_1^3(W_1)+I_1^2(W_3)$	$\cdots$	$-$
R_2	$I_2^1(W_1)$	$L_2^1(W_2)$	$\cdots$	$I_2^{N-1}(W_N)$	$L_2^2(W_2)+I_2^1(W_1)$	$-$	$\cdots$	$-$
R_3	$I_3^1(W_1)$	$I_3^2(W_2)$	$\cdots$	$I_3^{N-1}(W_N)$	$-$	$L_3^2(W_3)+I_3^1(W_1)$	$\cdots$	$-$
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
R_{N-1}	$I_{N-1}^1(W_1)$	$I_{N-1}^2(W_2)$	$\cdots$	$I_{N-1}^{N-1}(W_N)$	$-$	$-$	$\cdots$	$L_{N-1}^M(W_{N-1})+I_{N-1}^{N-1}(W_N)$
R_N	$I_N^1(W_1)$	$I_N^2(W_2)$	$\cdots$	$L_N^1(W_N)$	$-$	$-$	$\cdots$	$L_N^M(W_N)+I_N^{N-1}(W_{N-1})$

Table 4. Synergistic alternating CSIT states for $M \times N$ SISO X channel.

	Phase 1					Phase 2			
Time	1	2	3	$\cdots$	N	$N+1$	$N+2$	$\cdots$	$N+\dfrac{N(M-1)}{2}$
R_1	N	D	D	$\cdots$	D	P	P	$\cdots$	N
R_2	D	N	D	$\cdots$	D	P	N	$\cdots$	N
R_3	D	D	N	$\cdots$	D	N	P	$\cdots$	N
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
R_{N-1}	D	D	D	$\cdots$	D	N	N	$\cdots$	P
R_N	D	D	D	$\cdots$	N	N	N	$\cdots$	P

rescheduled transmission order. Also, Table 4 and its permutations are the minimum requirements of CSIT states for the proposed achievable scheme.

In Table 4, the numbers of P , D, and N states are $MN - N$, $N^2 - N$, and $N + N(N-2)(M-1)/2$, respectively. Since the total number of CSIT states is $N(N + N(M-1)/2)$, the CSIT distribution ratio $(\lambda_P, \lambda_D, \lambda_N)$ of the proposed scheme is $((2M-2)/(N(M+1)), (2N-2)/(N(M+1)), (MN-2M-N+4)/(N(M+1)))$. For $M = N = K$, as K increases, the DoF of the proposed scheme becomes 2 with the CSIT distribution ratio $(\lambda_P, \lambda_D, \lambda_N) = (0, 0, 1)$. It means that the DoF of the proposed scheme is asymptotically twice of that of no CSIT.

ii) $M \geq N$ when M is even and N is odd: In this case, the phase 1 is the same as that in the previous case but there are some modifications in the phase 2. For even M and odd N case, $(M-1)\binom{N}{2}/(N-1) = N(M-1)/2$ cannot be an integer. To achieve the same DoF as the previous case, the numbers of desired messages and the channel uses are doubled by denoting as $k = 1, 2$, i.e., $W_i^k = \{W_{ij}^k | j = 1, 2, \cdots, M\}$ is the desired messages for the receiver R_i and $2(N+N(M-1)/2)$ is the number of channel uses. The transmitted signals are given as in Table 5, where $W_i^k = \{W_{ij}^k | j = 1, 2, \cdots, M\}$ is the desired messages to the receiver R_i. All other procedures are the same as the previous case. For convenience, the precoding strategy and CSIT states are skipped. After the phase 2, during $2(N+N(M-1)/2)$ channel uses, each receiver has $2M$ linear combinations of its desired messages. Therefore, the N receivers can decode $2M$

desired messages by solving $2M$ linear equations and sum DoF $2MN/2(N+N(M-1)/2) = 2M/(M+1)$ is achieved by the proposed scheme.

iii) $N \geq M$ and even N: The overall situation of the scheme is the same but we consider minor modification in the phase 2. Achievable scheme is also composed of two separate phases described as below.

Phase 1) In this phase, N time slots are used. At time slot i, all transmitters transmit their desired messages to the receiver R_i, i.e., $X_j(i) = W_{ij}$, $1 \leq i \leq N$, $1 \leq j \leq M$.

Phase 2) In this phase, the transmitters send desired message pairs to the corresponding receiver pairs, where all pairs of desired messages assume to be disjoint. Thus, during $N/2$ time slots, each receiver receives one linear combination of its desired messages and interferences. After the phase 1, each receiver obtains one linear combination of the desired messages. Therefore, each receiver needs $M-1$ additional linear combinations of M desired messages. For that purpose, all transmitters transmit repeatedly until all receivers have received $M-1$ linear combinations of the desired messages in the phase 2. Let α be the repeated number of $N/2$ transmissions in the phase 2. Since each receiver obtains one linear combinations of its desired messages during one repetition, the repeated number α should be $\alpha = M - 1$. Then, the total number of transmissions in the phase 2 is equal to $(M-1)N/2$. The transmitted messages are precoded for interference cancellation using the previous time slot CSI. The transmitted signals are given as in Table 6, where

Table 5. Transmitted signals for $M \geq N$, when M is even and N is odd.

	Phase 1				Phase 2					
Time	1	2	$\cdots$	$2N$	$2N+1$	$2N+2$	$\cdots$	$2(N+\binom{N}{2})$	$\cdots$	$2(N+\frac{N(M-1)}{2})$
Tx	W_1^1	W_2^1	$\cdots$	W_N^2	W_1^1, W_2^1	W_1^1, W_3^1	$\cdots$	W_{N-1}^2, W_N^2	$\cdots$	W_{N-1}^2, W_N^2

Table 6. Transmitted signals for $N \geq M$ and even N.

	Phase 1				Phase 2					
Time	1	2	$\cdots$	N	$N+1$	$N+2$	$\cdots$	$N+\frac{N}{2}$	$\cdots$	$N+(M-1)\frac{N}{2}$
Tx	W_1	W_2	$\cdots$	W_N	W_1, W_2	W_3, W_4	$\cdots$	W_{N-1}, W_N	$\cdots$	W_{N-1}, W_N

$W_i = \{W_{ij}|j = 1, 2, \cdots, M\}$ is the desired messages for the receiver R_i.

All other procedures are the same as the previous case. For convenience, the precoding strategy and CSIT states are skipped. After the phase 2, during $N + (M - 1)N/2$ channel uses, each receiver has M linear combinations of its desired messages. Therefore, the N receivers can decode M desired messages by solving M linear equations and DoF $MN/(N + (M - 1)N/2) = 2M/(M + 1)$ is achieved by the proposed scheme.

iv) $N \geq M$ and odd N: Similar to the case ii), the numbers of desired messages and channel uses are doubled by denoting as k=1,2, i.e., $W_i^k = \{W_{ij}^k|j = 1, 2, \cdots, M\}$ is the desired messages for the receiver R_i and $2(N+(M-1)N/2)$ is the number of channel uses. The transmitted signals are given as in Table 7, where $W_i^k = \{W_{ij}^k|j = 1, 2, \cdots, M\}$ is the desired messages to the receiver R_i.

All other procedures are the same as the previous case. For convenience, the precoding strategy and CSIT states are skipped. After the phase 2, during $2(N + (M - 1)N/2)$ channel uses, each receiver has $2M$ linear combinations of its desired messages. Therefore, the N receivers can decode $2M$ desired messages by solving $2M$ linear equations and DoF $2MN/2(N + (M - 1)N/2) = 2M/(M + 1)$ is achieved by the proposed scheme.

Theorem 1: The achievable DoF for $M \times N$ SISO X channel with synergistic alternating CSIT are given as

$$\text{DoF}_{sum} = \frac{2M}{M + 1}. \tag{13}$$

As a result, the proposed schemes achieve $2M/(M + 1)$ DoF for $M \times N$ SISO X channel with synergistic alternating CSIT.

IV. SYNERGISTIC GAIN

Since there is no conventional scheme that uses the same CSI feedback, we show that the DoF values of the proposed alternating CSIT setting can be strictly larger than a weighted sum of the DoF values of the conventional schemes with the same CSIT settings. We call this the synergistic gain of alternating CSIT and the benefits of alternating CSIT over non alternating CSIT can be quite substantial.

In order to derive the synergistic gain of the proposed scheme, we compare its DoF with the conventional schemes without

synergistic alternating CSIT. For the conventional schemes, the achievable DoF can be computed as the weighted sum of individual DoFs for P, D, and N states [2], [4]. When $M = N = K$, the CSIT distribution ratio $(\lambda_P, \lambda_D, \lambda_N)$ of the proposed scheme is $((2K - 2)/(K^2 + K), (2K - 2)/(K^2 + K), (K^2 - 3K + 4)/(K^2 + K))$ as in Table 4. Therefore, the weighted sum of individual DoFs for the conventional schemes is given as

$$\text{DoF}_{ws}(K) = \frac{2K - 2}{K^2 + K}\text{DoF}_{perfect}(K) + \frac{2K - 2}{K^2 + K}\text{DoF}_{delayed}(K)$$
$$+ \frac{K^2 - 3K + 4}{K^2 + K}\text{DoF}_{no}(K). \tag{14}$$

It is known that $\text{DoF}_{delayed}(K) = 4/3 - 2/(3(3K - 1))$ and $\text{DoF}_{no}(K) = 1$ for the finite time extension, but for the perfect CSIT, $\text{DoF}_{perfect}(K) = K^2/(2K - 1)$ for the infinite time extension. If we consider only the finite time extension, (14) can be modified as

$$\text{DoF}_{ws}(K) \leq \frac{2K - 2}{K^2 + K}\frac{K^2}{2K - 1} + \frac{2K - 2}{K^2 + K}\left(\frac{4}{3} - \frac{2}{(3(3K - 1))}\right)$$
$$+ \frac{K^2 - 3K + 4}{K^2 + K}. \tag{15}$$

As shown in Fig. 2, the proposed scheme using synergistic alternating states gives the larger DoF than the upper bound of the weighted sum DoF for the conventional schemes. This DoF gain is called synergistic gain.

For the general M, N cases, we can obtain the DoF gain in the same way as for K-user case. The CSIT distribution ratio $(\lambda_P, \lambda_D, \lambda_N)$ of the proposed scheme is $((2M - 2)/(N(M + 1)), (2N - 2)/(N(M + 1)), (MN - 2M - N + 4)/(N(M + 1)))$. Therefore, the weighted sum of individual DoFs for the conventional schemes is given as

$$\text{DoF}_{ws}(M, N) = \frac{2M - 2}{N(M + 1)}\text{DoF}_{perfect}(M, N)$$
$$+ \frac{2N - 2}{N(M + 1)}\text{DoF}_{delayed}(M, N)$$
$$+ \frac{MN - 2M - N + 4}{N(M + 1)}\text{DoF}_{no}(M, N). \tag{16}$$

It is also known that $\text{DoF}_{perfect}(M, N) = MN/(M + N - 1)$ for the infinite time extension. Since the achievable DoF for the $M \times N$ X channel with delayed CSIT is not known, we can obtain upper bound on the $\text{DoF}_{delayed}(M, N)$ from approximation

Table 7. Transmitted signals for $N \geq M$ and odd N.

	Phase 1				Phase 2					
Time	1	2	$\cdots$	$2N$	$2N+1$	$2N+2$	$\cdots$	$2(N+\frac{N}{2})$	$\cdots$	$2(N+(M-1)\frac{N}{2})$
Tx	W_1^1	W_2^1	$\cdots$	W_N^2	W_1^1, W_2^1	W_3^1, W_4^1	$\cdots$	W_{N-1}^2, W_N^2	$\cdots$	W_{N-1}^2, W_N^2

Fig. 2. Comparison of DoF of the proposed scheme and the weighted sum DoF of the conventional scheme.
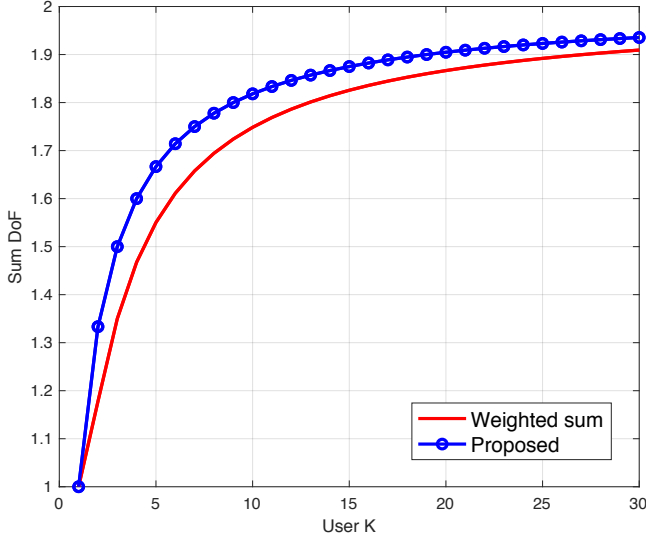

Fig. 3. Comparison of DoF of the proposed scheme and the weighted sum DoF of the conventional scheme.

of $\text{DoF}_{\text{delayed}}(K)$. When $M = N + a$, we add a receivers and construct M-user X channel or remove a transmitters and construct N-user X channel. Then, $\text{DoF}_{\text{delayed}}(M, N)$ is bounded as

$$\text{DoF}_{\text{delayed}}(N) \leq \text{DoF}_{\text{delayed}}(M, N) \leq \text{DoF}_{\text{delayed}}(M). \quad (17)$$

When $N = M + a$, we can also obtain upper-bound of $\text{DoF}_{\text{delayed}}(M, N)$ in the same manner. Therefore, the achievable DoF for the $M \times N$ X channel with delayed CSIT is bounded as

$$\text{DoF}_{\text{delayed}}(M, N) \leq \frac{4}{3} - \frac{2}{(3(3\max(M, N) - 1))}. \quad (18)$$

If we consider only the finite time extension, (16) can be modified as

$$
\begin{aligned}
\text{DoF}_{\text{ws}}(M, N) \leq & \frac{2M - 2}{N(M+1)} \frac{MN}{(M+N-1)} \\
& + \frac{2N - 2}{N(M+1)} \left(\frac{4}{3} - \frac{2}{(3(3\max(M, N) - 1))} \right) \\
& + \frac{MN - 2M - N + 4}{N(M+1)}.
\end{aligned} \quad (19)
$$

In Fig. 3, DoFs for the proposed scheme, $2M/(M+1)$ and the weighted sum DoF in (19) are given. In the general M, N cases, the synergistic gain exists against the upper bound of the weighted sum DoF only for $M \leq N$ because DoF of the proposed scheme is not a function of N.
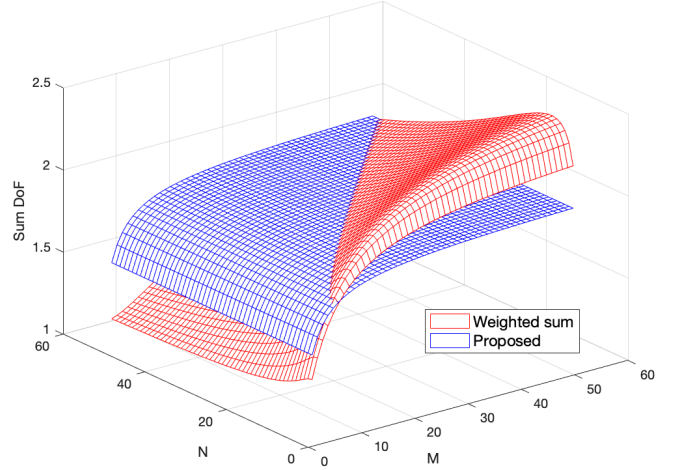
V. CONCLUSION

We propose the achievable scheme of the $M \times N$ SISO X channel with synergistic alternating CSIT and derive its DoF $2M/(M+1)$. The synergistic gain for the alternating CSIT still exists in the $M \times N$ SISO X channel. Sum DoF $2M/(M+1)$ implies that if the number of transmitters is fixed to M, the sum DoF is determined by M and each receiver has DoF divided by the total number of receivers. In the proposed scheme for the $M \times N$ SISO X channel, the achievable DoF for $M \times N$ X channel does not scale with the number of users unlike the DoF with the perfect and instantaneous CSIT.

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