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Multi-Attribute Decision-Making Method Applying a Novel Correlation Coefficient of Interval-Valued Neutrosophic Hesitant Fuzzy Sets

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Abstract

Interval-valued neutrosophic hesitant fuzzy set (IVNHFS) is an extension of neutrosophic set (NS) and hesitant fuzzy set (HFS), each element of which has truth membership hesitant function, indeterminacy membership hesitant function and falsity membership hesitant function and the values of these functions lie in several possible closed intervals in the real unit interval [0,1]. In contrast with NS and HFS, IVNHFS can be more flexibly used to deal with uncertain, incomplete, indeterminate, inconsistent and hesitant information. In this study, I propose the novel correlation coefficient of IVNHFSs and my paper discusses its properties. Then, based on the novel correlation coefficient, I develop an approach to deal with multi-attribute decision-making problems within the framework of IVNHFS. In the end, a practical example is used to show that the approach is reasonable and effective in dealing with decision-making problems.

Keywords

Correlation Coefficient, Decision-Making, Interval-Valued Neutrosophic Hesitant Fuzzy Set

1. Introduction

In 1965, Zadeh [1] put forward the fuzzy set which used membership degree to describe the uncertain information in the real world. Since the fuzzy set was raised, researchers have made many achievements and it has been widely applied in decision-making, clustering analysis, image processing, pattern recognition, information fusion, etc. Based on Zadeh's research, many extensions of fuzzy set have been developed, such as the interval-valued fuzzy set [2], L-fuzzy set [3], (interval-valued) intuitionistic fuzzy set [4,5], hesitant fuzzy set (HFS) [6] and neutrosophic set (NSs) [7]. Due to the uncertainty and complexity of the society, the growing amount of decision information and alternatives make the decision making problems difficult. Decision makers have to find the proper method to deal with the multi-attribute decision-making (MADM) problems and choose the best alternative. Aggregation operators and measures (entropy, similarity measure, and correlation coefficient) are two important methods that researchers pay more attention to in handling the MADM problems.

One extension of the fuzzy set is the NS which was proposed by Smarandache [7]. The NS is more

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accurate and flexible to express the incomplete, indeterminate and inconsistent information than the intuitionistic fuzzy set and each element of which is made up of three membership degrees, the truth membership degree, the falsity membership degree and the indeterminacy membership degree, whose values lie within non-standard or standard intervals in NS theory. In the last 10 years, many fruitful results have been achieved within the framework of NS. Later, Wang simplified the range of the values in standard single values/intervals and developed the single valued NS (SVNS) and the interval-valued NS (IVNS) [8,9]. Ye [10] developed the simplified neutrosophic set (SNS) which is the generalization of SVNS and IVNS, and gave some operations and aggregation operators on SNS. Liu and Teng [11] proposed the normal neutrosophic power aggregation operators and studied their properties, then used them for decision-making problems. Ye [12] put forward the correlation coefficient of SVNSs and discussed its properties, then used it for decision-making conditions. Under the environment of NS, Liu and Luo [13] gave the weighted distance measure based method to handle group decision-making problems with unknown attribute weights. The other extension of the fuzzy set is the HFS which was presented by Torra [6]. It is a powerful tool to deal with uncertain and hesitant information and its membership degrees have several possible values in the real unit interval [0,1]. The authors [14,15] have achieved a lot of results in this respect. Chen et al. [16] proposed the interval-valued HFS (IVHFS) which extends the membership degrees from single values to interval values. With the development of these theories, Wang and Li [17] put forward the multi-valued NS (MVNS) which has the advantages of NS and HFS. Peng et al. [18] studied the ELECTRE method to deal with decision-making problems under the conditions of MVNS. Liu and Shi [19] proposed the interval-valued neutrosophic hesitant fuzzy set (IVNHFS) which is the extension of IVHFS and IVNS, and its elements have a few closed intervals in the real unit interval [0,1] to better depict the uncertain information.

As an important statistical measure, correlation coefficient has been extended to the fuzzy set which reflects how the two sets move in relation to each other. The bigger the correlation coefficient is, the closer the relationship between the two sets is. Many researchers have studied the correlation coefficients of different kinds of sets and discussed their properties. Based on the correlation coefficients, some decision-making methods and clustering analysis methods are studied. Karaaslan [20] proposed correlation coefficients of single-valued neutrosophic refined soft sets and applied them to clustering analysis, Shi [21] developed correlation coefficient of SNS and used it for bearing fault diagnosis. Sahin and Liu [22] proposed correlation coefficients of single-valued neutrosophic HFS. Meng and Chen [23] developed correlation coefficients of HFS and applied them to clustering analysis and decision-making. Ye [24] proposed correlation coefficients of dual HFS and applied them to decision-making. Depending on the existing correlation coefficients, I propose a novel correlation coefficient of IVNHFSs and discuss its properties. Then a MADM method within the framework of IVNHFS is studied based on the novel correlation coefficient. At last, I apply the proposed decision-making method to deal with the investment problem.

The paper is organized as follows: in Section 2, I recall the basic concepts of IVNS, IVHFS, IVNHFS. In Section 3, a novel correlation coefficient of IVNHFSs is developed. At the same time, its properties are studied. In Section 4, a MADM method is proposed on the basis of the correlation coefficient of IVNHFSs. In Section 5, I use an example to illustrate the feasibility of the proposed MADM method. In the end, I give the conclusion of my study in Section 6.

2. Basic Definitions

In this section, I will review the basic knowledge of IVNS, IVHFS, IVNHFS required in the following part.

2.1 Interval-Valued Neutrosophic Set

The IVNS extends the membership functions of the NS from exact numbers to intervals which make NS better to describe the uncertainty information of the real world. Assume *X* is a universe of discourse and *x* be a generic element in *X*. An interval-valued neutrosophic set *A* on *X* is denoted by

$$A = \{< x, T_A(x), I_A(x), F_A(x) > | \, x \in X\}$$

where $T_A(x)$, $I_A(x)$ and $F_A(x)$ are the truth membership function, indeterminacy membership function and falsity membership function, respectively, and their values lie in the closed intervals in the real unit interval [0, 1] and represent the decision maker's preference for the attribute of different alternatives. Here $T_A(x), I_A(x), F_A(x) \subseteq [0,1]$.

2.2 Interval-Valued Hesitant Fuzzy Set

The IVHFS is the extension of the HFS that depicts people's hesitancy in their preference over the decision making problems. Assume *X* be a universe of discourse, an IVHFS *A* on *X* is defined as:

$$\tilde{A} = \left\{ \left\langle x, h_{\tilde{A}}(x) \right\rangle \middle| x \in X \right\}$$

where $h_{\tilde{A}}(x)$ is a set of some closed intervals in the real unit interval [0,1], and represents the possible membership degree of the element *x* to *A* and reflects the decision maker's hesitant information.

2.3 Interval-Valued Neutrosophic Hesitant Fuzzy Set

As a generalization of FS, IVNHFS is a combination of IVNS and IVHFS which is a powerful tool to process the uncertain, incomplete, inconsistent and hesitant information [19]. Assume *X* is a non-empty finite set, an IVNHFS *N* on *X* is defined as:

$$N = \left\{ \left\langle x, \tilde{t}(x), \tilde{i}(x), \tilde{f}(x) \right\rangle \middle| x \in X \right\}$$

where $\tilde{t}(x) = \left\{ \tilde{\gamma} \mid \tilde{\gamma} \in \tilde{t}(x) \right\}$, $\tilde{i}(x) = \left\{ \tilde{\delta} \mid \tilde{\delta} \in \tilde{i}(x) \right\}$ and $\tilde{f}(x) = \left\{ \tilde{\eta} \mid \tilde{\eta} \in \tilde{f}(x) \right\}$ are three membership functions expressed by a few closed intervals in the real unit interval [0,1] which represent the truth membership hesitant degree, indeterminacy membership hesitant degree and falsity membership hesitant degree and meet the following conditions:

$$\tilde{\gamma} = \left[\gamma^L, \gamma^U \right] \subseteq [0,1], \ \tilde{\delta} = \left[\delta^L, \delta^U \right] \subseteq [0,1], \ \tilde{\eta} = \left[\eta^L, \eta^U \right] \subseteq [0,1]$$

Specially, if the set X has only one element x, we call the set $N=\left\{\tilde{t}(x),\tilde{i}(x),\tilde{f}(x)\right\}$ an interval-valued neutrosophic hesitant fuzzy element (IVNHFE). For convenience, we abbreviate it as the symbol $N=\left\{\tilde{t},\tilde{i},\tilde{f}\right\}$.

3. A Novel Correlation Coefficient of IVNHFSs

Based on the research work [25], I find that the computational process is very complex which is very difficult to calculate the correlation coefficient of IVNHFSs. To reduce the difficulties, in this section, I will give a novel formula of correlation coefficient of IVNHFSs.

Let A be an IVNHFS on a reference set $X=\left\{x_1,x_2,\cdots x_n\right\}$ and denoted as:

$$A=\left\{\left\langle x_i,\tilde{t}(x_i),\tilde{i}(x_i),\tilde{f}(x_i)\right\rangle \middle| x_i\in X\right\}$$

we call

$$E_{\mathrm{IVNHFS}}(A)=\left(\gamma^L\right)^2+\left(\gamma^U\right)^2+\left(\delta^L\right)^2+\left(\delta^U\right)^2+\left(\eta^L\right)^2+\left(\eta^U\right)^2 \tag{1}$$

the informational energy of A.
where

$$\gamma^L=\min\left\{\gamma_1^L,\gamma_2^L,\cdots,\gamma_k^L\right\},\ \gamma^U=\max\left\{\gamma_1^U,\gamma_2^U,\cdots,\gamma_k^U\right\}$$

$$\delta^L=\min\left\{\delta_1^L,\delta_2^L,\cdots,\delta_l^L\right\},\ \delta^U=\max\left\{\delta_1^U,\delta_2^U,\cdots,\delta_l^U\right\}$$

$$\eta^L=\min\left\{\eta_1^L,\eta_2^L,\cdots,\eta_m^L\right\},\ \eta^U=\max\left\{\eta_1^U,\eta_2^U,\cdots,\eta_m^U\right\}$$

And k is the number of values in $\tilde{t}$, l is the number of values in $\tilde{i}$, m is the number of values in $\tilde{f}$.

The informational energy is the information contained in the IVNHFS A. In the following, I will give the informational energy of A and B, which I define the correlation of A and B which responds the relationship between the two sets.

Suppose A and B are two IVNHFSs on a reference set $X=\left\{x_1,x_2,\cdots x_n\right\}$ and I denote them as

$$A=\left\{\left\langle x_i,\tilde{t}_A(x_i),\tilde{i}_A(x_i),\tilde{f}_A(x_i)\right\rangle \middle| x_i\in X\right\},$$

$$B=\left\{\left\langle x_i,\tilde{t}_B(x_i),\tilde{i}_B(x_i),\tilde{f}_B(x_i)\right\rangle \middle| x_i\in X\right\}$$

respectively, and

$$\tilde{t}_A(x_i)=\left\{\tilde{\gamma}_{Aj}\mid\tilde{\gamma}_{Aj}\in\tilde{t}_A(x_i)\right\},$$

$$\tilde{i}_A(x_i)=\left\{\tilde{\delta}_{Aj}\mid\tilde{\delta}_{Aj}\in\tilde{i}_A(x_i)\right\},$$

$$\tilde{f}_A(x_i)=\left\{\tilde{\eta}_{Aj}\mid\tilde{\eta}_{Aj}\in\tilde{f}_A(x_i)\right\},$$

where

$$\tilde{\gamma}_{Aj}=\left[\gamma_{Aj}^{L},\gamma_{Aj}^{U}\right]\subseteq[0,1],\ j=1,2,\cdots k_{1},$$
$$\tilde{\delta}_{Aj}=\left[\delta_{Aj}^{L},\delta_{Aj}^{U}\right]\subseteq[0,1],\ j=1,2,\cdots l_{1},$$
$$\tilde{\eta}_{Aj}=\left[\eta_{Aj}^{L},\eta_{Aj}^{U}\right]\subseteq[0,1],\ j=1,2,\cdots m_{1},$$

and

$$\tilde{t}_{B}(x_{i})=\left\{\tilde{\gamma}_{Bs}\mid\tilde{\gamma}_{Bs}\in\tilde{t}_{B}(x_{i})\right\},$$
$$\tilde{i}_{B}(x_{i})=\left\{\tilde{\delta}_{Bs}\mid\tilde{\delta}_{Bs}\in\tilde{i}_{B}(x_{i})\right\},$$
$$\tilde{f}_{B}(x_{i})=\left\{\tilde{\eta}_{Bs}\mid\tilde{\eta}_{Bs}\in\tilde{f}_{B}(x_{i})\right\},$$

where

$$\tilde{\gamma}_{Bs}=\left[\gamma_{Bs}^{L},\gamma_{Bs}^{U}\right]\subseteq[0,1],\ s=1,2,\cdots k_{2},$$
$$\tilde{\delta}_{Bs}=\left[\delta_{Bs}^{L},\delta_{Bs}^{U}\right]\subseteq[0,1],\ s=1,2,\cdots l_{2},$$
$$\tilde{\eta}_{Bs}=\left[\eta_{Bs}^{L},\eta_{Bs}^{U}\right]\subseteq[0,1],\ j=1,2,\cdots m_{2}$$

Afterwards, the correlation of A and B is defined as

$$C_{\mathrm{IVNHFS}}(A,B)=\gamma_{A}^{L}\gamma_{B}^{L}+\gamma_{A}^{U}\gamma_{B}^{U}+\delta_{A}^{L}\delta_{B}^{L}+\delta_{A}^{U}\delta_{B}^{U}+\eta_{A}^{L}\eta_{B}^{L}+\eta_{A}^{U}\eta_{B}^{U} \tag{2}$$

where

$$\gamma_{A}^{L}=\min\left\{\gamma_{A1}^{L},\gamma_{A2}^{L},\cdots,\gamma_{Ak_{1}}^{L}\right\},\ \gamma_{A}^{U}=\max\left\{\gamma_{A1}^{U},\gamma_{A2}^{U},\cdots,\gamma_{Ak_{1}}^{U}\right\},$$
$$\delta_{A}^{L}=\min\left\{\delta_{A1}^{L},\delta_{A2}^{L},\cdots,\delta_{Al_{1}}^{L}\right\},\ \delta_{A}^{U}=\max\left\{\delta_{A1}^{U},\delta_{A2}^{U},\cdots,\delta_{Al_{1}}^{U}\right\},$$
$$\eta_{A}^{L}=\min\left\{\eta_{A1}^{L},\eta_{A2}^{L},\cdots,\eta_{Am_{1}}^{L}\right\},\ \eta_{A}^{U}=\max\left\{\eta_{A1}^{U},\eta_{A2}^{U},\cdots,\eta_{Am_{1}}^{U}\right\},$$
$$\gamma_{B}^{L}=\min\left\{\gamma_{B1}^{L},\gamma_{B2}^{L},\cdots,\gamma_{Bk_{2}}^{L}\right\},\ \gamma_{B}^{U}=\max\left\{\gamma_{B1}^{U},\gamma_{B2}^{U},\cdots,\gamma_{Bk_{2}}^{U}\right\},$$
$$\delta_{B}^{L}=\min\left\{\delta_{B1}^{L},\delta_{B2}^{L},\cdots,\delta_{Bl_{2}}^{L}\right\},\ \delta_{B}^{U}=\max\left\{\delta_{B1}^{U},\delta_{B2}^{U},\cdots,\delta_{Bl_{2}}^{U}\right\},$$
$$\eta_{B}^{L}=\min\left\{\eta_{B1}^{L},\eta_{B2}^{L},\cdots,\eta_{Bm_{2}}^{L}\right\},\ \eta_{B}^{U}=\max\left\{\eta_{B1}^{U},\eta_{B2}^{U},\cdots,\eta_{Bm_{2}}^{U}\right\},$$

It is easy to get the following result:

$$C_{\mathrm{IVNHFS}}(A,A)=E_{\mathrm{IVNHFS}}(A),$$
$$C_{\mathrm{IVNHFS}}(A,B)=C_{\mathrm{IVNHFS}}(B,A).$$

On the basis of the definition of informational energy and the correlation, I construct the correlation coefficient of two IVNHFSs. I call the correlation coefficient of A and B.

$$\lambda_{\mathrm{IVNHFS}}(A,B)=\frac{C_{\mathrm{IVNHFS}}(A,B)}{\left[C_{\mathrm{IVNHFS}}(A,A)\right]^{\frac{1}{2}}\left[C_{\mathrm{IVNHFS}}(B,B)\right]^{\frac{1}{2}}} \tag{3}$$

Computation Analysis: The advantage of Eq. (3) is that the computation is simpler than Ye's equation in [25]. The first step is to choose the minimum and the maximum values of $\tilde{t}$, $\tilde{i}$, $\tilde{f}$. Then we change the IVNHFS to interval-valued neutrosophic set, that is to say, several closed intervals change to one closed intervals. It can be seen as the simple form of IVNHFS that can broadly depict the uncertain information.

Let A and B be two IVNHFSs, the novel correlation coefficient of IVNHFSs A and B has three properties:

[1] $\lambda_{\text{IVNHFS}}(A,B)=\lambda_{\text{IVNHFS}}(B,A)$;

[2] $0\le\lambda_{\text{IVNHFS}}(A,B)\le 1$;

[3] $\lambda_{\text{IVNHFS}}(A,B)=1$, if $A=B$.

Proof. [1] and [3] are easily obtained by definition of correlation coefficient, we need prove [2].

According to the Cauchy-Schwarz inequality,

$$\left(x_1y_1+x_2y_2+\cdots x_ny_n\right)^2\le\left(x_1^2+x_2^2+\cdots x_n^2\right)\left(y_1^2+y_2^2+\cdots y_n^2\right)$$

We obtain

$$\begin{aligned}C_{\text{IVNHFS}}(A,B)&=\left(\gamma_A^L\gamma_B^L+\gamma_A^U\gamma_B^U+\delta_A^L\delta_B^L+\delta_A^U\delta_B^U+\eta_A^L\eta_B^L+\eta_A^U\eta_B^U\right)\\&\le\left[\left(\gamma_A^L\right)^2+\left(\gamma_A^U\right)^2+\left(\delta_A^L\right)^2+\left(\delta_A^U\right)^2+\left(\eta_A^L\right)^2+\left(\eta_A^U\right)^2\right]^{\frac{1}{2}}\\&\quad\left[\left(\gamma_B^L\right)^2+\left(\gamma_B^U\right)^2+\left(\delta_B^L\right)^2+\left(\delta_B^U\right)^2+\left(\eta_B^L\right)^2+\left(\eta_B^U\right)^2\right]^{\frac{1}{2}}\end{aligned}$$

Therefore, $C_{\text{IVNHFS}}(A,B)\le\left[C_{\text{IVNHFS}}(A,A)\right]^{\frac{1}{2}}\left[C_{\text{IVNHFS}}(B,B)\right]^{\frac{1}{2}}$.

Thus I get $0\le\lambda_{\text{IVNHFS}}(A,B)\le 1$.

4. Multi-Attribute Decision-Making Method based on a Novel Correlation Coefficient

I will use the IVNHFS numbers to denote the MADM information. Suppose there are m alternatives, and each alternative has n attributes. For convenience, we denote them as $X=\{x_1,x_2,\cdots x_m\}$ and $C=\{c_1,c_2,\cdots c_n\}$, respectively. In MADM problems, the ideal point offers a feasible method to evaluate the best alternative from the set of alternatives since it is virtual in the world. In this part, I also use the ideal point to identify the best alternative in the decision set. Here I assume the ideal IVNHFE as $A^{\#}=\{\tilde{t}^*,\tilde{i}^*,\tilde{f}^*\}=\{\{[1,1],[1,1],[1,1]\},\{[0,0],[0,0],[0,0]\},\{[0,0],[0,0],[0,0]\}\}$.

Then, the simplification of $A^{\#}$ is $A^*=\{[1,1],[0,0],[0,0]\}$.

I utilize the proposed novel correlation coefficient to develop a method to deal with MADM problem within the conditions of IVNHFSs.

Let

$$A_{ij}=\left\{\begin{array}{l}\left\{\left[\gamma_{A_{ij}^1}^L,\gamma_{A_{ij}^1}^U\right],\cdots,\left[\gamma_{A_{ij}^k}^L,\gamma_{A_{ij}^k}^U\right]\right\},\left\{\left[\delta_{A_{ij}^1}^L,\delta_{A_{ij}^1}^U\right],\cdots,\left[\delta_{A_{ij}^l}^L,\delta_{A_{ij}^l}^U\right]\right\},\\ \left\{\left[\eta_{A_{ij}^1}^L,\eta_{A_{ij}^1}^U\right],\cdots,\left[\eta_{A_{ij}^m}^L,\eta_{A_{ij}^m}^U\right]\right\}\end{array}\right\}$$

denote the decision information, and $A_i=\left\{\left[\gamma_{A_i}^L,\gamma_{A_i}^U\right],\left[\delta_{A_i}^L,\delta_{A_i}^U\right],\left[\eta_{A_i}^L,\eta_{A_i}^U\right]\right\}$ be the simplification of A_{ij}, where

$$\gamma_{A_i}^L=\min\left\{\gamma_{A_{i1}}^L,\gamma_{A_{i2}}^L,\cdots,\gamma_{A_{ik}}^L\right\},\gamma_{A_i}^U=\min\left\{\gamma_{A_{i1}}^U,\gamma_{A_{i2}}^U,\cdots,\gamma_{A_{ik}}^U\right\},$$

$$\delta_{A_i}^L=\min\left\{\delta_{A_{i1}}^L,\delta_{A_{i2}}^L,\cdots,\delta_{A_{il}}^L\right\},\delta_{A_i}^U=\min\left\{\delta_{A_{i1}}^U,\delta_{A_{i2}}^U,\cdots,\delta_{A_{il}}^U\right\},$$

$$\eta_{A_i}^L=\min\left\{\eta_{A_{i1}}^L,\eta_{A_{i2}}^L,\cdots,\eta_{A_{im}}^L\right\},\eta_{A_i}^U=\min\left\{\eta_{A_{i1}}^U,\eta_{A_{i2}}^U,\cdots,\eta_{A_{im}}^U\right\}.$$

The process is as follows：

Step 1. Calculate the novel correlation coefficient $\lambda_{\text{IVNHFS}}\left(A_i,A^*\right)$ according to Eqs. (1), (2), and (3).

Step 2. According to the values of the novel correlation coefficient, I rank the alternatives. The bigger the correlation coefficient is, the better the alternative to the ideal element is.

Step 3. Choose the best alternative in accordance with the maximum value of correlation coefficients.

5. Numerical Example and Analysis

5.1 Numerical Example

In this section, a practical decision making example adapted from [25] is used to clarify the feasibility of the proposed MADM method. There is a financial company that wants to make money from the following four potential alternatives: A_1, a transportation company; A_2, a decoration company; A_3, a beauty salon company; and A_4, an education training company. The decision maker of the financial company must make a decision according to the attributes of each alternative: C_1 (the potential hazard analysis); C_2 (the adolescence analysis); C_3 (the surrounding impact analysis). According to the three attributes, the expert gave the decision information according to his knowledge background and the characteristic of the alternatives and described by the interval-valued neutrosophic hesitant fuzzy elements that is listed in Table 1.

Table 1. Interval-valued neutrosophic hesitant fuzzy information

	C_1	C_2	C_3
A_1	{{[0.3,0.4],[0.4,0.4], [0.4,0.5]}, {[0.1,0.2]},{[0.3,0.4]}}	{{[0.4,0.5],[0.5,0.6]}, {[0.2,0.3]}, {[0.3,0.3],[0.3,0.4]}}	{{[0.2,0.3]}, {[0.1,0.2]}, {[0.4,0.5],[0.5,0.6]}}
A_2	{{[0.6,0.7]},{[0.1,0.2]}, {[0.2,0.3]}}	{{[0.5,0.7]}, {[0.1,0.2]}, {[0.2,0.3]}}	{{[0.6,0.7]},{[0.1,0.2]}, {[0.1,0.2]},
A_3	{{[0.3,0.4], [0.5,0.6]}, {[0.2,0.4]}, {[0.2,0.3]}}	{{[0.5,0.6]}, {[0.2,0.3]}, {[0.3,0.4]}}	{{[0.5,0.6]}, {[0.1,0.2], [0.2,0.3]}, {[0.2,0.3]}}
A_4	{{[0.7,0.8]}, {[0,0.1]}, {[0.1,0.2]}}	{{[0.6,0.7]}, {[0,0.1]}, {[0.2,0.3]}}	{{[0.3,0.5]}, [0.2,0.3]}, {[0.1,0.2],[0.3,0.4]}}

According to Section 4, the decision-making process is as follows.

First, I calculate the novel correlation coefficient $\lambda_{\text{IVNHFS}}\left(A_i, A^*\right)\left(i=1,2,3,4\right)$ according to Eq. (1), (2), (3) as follows:

$$\lambda_{\text{IVNHFS}}\left(A_1, A^*\right)=0.5804,$$
$$\lambda_{\text{IVNHFS}}\left(A_2, A^*\right)=0.9192,$$
$$\lambda_{\text{IVNHFS}}\left(A_3, A^*\right)=0.7028,$$
$$\lambda_{\text{IVNHFS}}\left(A_4, A^*\right)=0.7817$$

Since

$$\lambda_{\text{IVNHFS}}\left(A_2, A^*\right)\geq\lambda_{\text{IVNHFS}}\left(A_4, A^*\right)\geq\lambda_{\text{IVNHFS}}\left(A_3, A^*\right)\geq\lambda_{\text{IVNHFS}}\left(A_1, A^*\right)$$

Then, I rank the alternative as $A_2 \succ A_4 \succ A_3 \succ A_1$. The best alternative is A_2, that is, a decoration company is the best alternative.

5.2 Analysis

In this part, I have proposed a new method to deal with the MADM problem described by IVNHFS information. In the MADM problems, I adopt the novel correlation coefficient of IVNHFSs to rank the alternatives and obtain the best one. Moreover, the calculation of the method is simpler and more practical. In order to show the feasibility of my approach, I will compare the method of [19] and [25] with the proposed method in this paper.

Aggregation Operator Method [19]: They use the interval-valued neutrosophic hesitant fuzzy generalized hybrid weighted aggregation (INHFGHWA) operators to deal with the MADM problems. First, they aggregate the data of different attributes of the alternative based on Eq. (47) in Reference [19]. Then, they calculate the score functions of the alternatives as follows:

$$s\left(A_1\right)=1.0273, s\left(A_2\right)=1.402, s\left(A_3\right)=1.1339, s\left(A_4\right)=1.3969.$$

So we rank the alternatives as $A_2 \succ A_4 \succ A_3 \succ A_1$, then A_2 is the best alternative.

Method of [25]： They use the interval-valued neutrosophic hesitant fuzzy correlation coefficient to deal with the MADM problems. They use Eq. (7) in [25] to calculate the correlation coefficient as follows:

$$\lambda_{\text{IVNHFS}}\left(A_1, A^*\right)=0.4766, \lambda_{\text{IVNHFS}}\left(A_2, A^*\right)=0.9285,$$
$$\lambda_{\text{IVNHFS}}\left(A_3, A^*\right)=0.6823, \lambda_{\text{IVNHFS}}\left(A_4, A^*\right)=0.9053$$

Since $\lambda_{\text{IVNHFS}}\left(A_2, A^*\right)\geq\lambda_{\text{IVNHFS}}\left(A_4, A^*\right)\geq\lambda_{\text{IVNHFS}}\left(A_3, A^*\right)\geq\lambda_{\text{IVNHFS}}\left(A_1, A^*\right)$, and I rank the alternative as $A_2 \succ A_4 \succ A_3 \succ A_1$. The most desirable one is A_2, which is agreement with the result of the proposed method.

From the above analysis, I find that three ways have the same result that the best alternative is A_2. First, let us consider the method of [19], the decision making method based on the INHFGHWA operators used the score functions to rank the alternatives. It dealt with the problem using the aggregation operators and fully used the decision information values that the expert gave. I found that the calculation is very complex. In [25], the author gave several correlation coefficient formulas, the calculation of the correlation coefficient is complex. With regard to my proposed method, the ideal element is defined first, my method is based on the novel formula. I used the IVNHFS to get a new IVNS, and got the novel correlation coefficient of IVNHFSs, the calculation is simple and effective. In a word, the proposed method in my paper gave a new approach to calculate the correlation coefficient and provide an alternative perspective in dealing with MADM problems.

6. Conclusions

In the study, a novel correlation coefficient of IVNHFSs is proposed and its properties are discussed. On the basis of the novel correlation coefficient, I have developed a method to cope with the MADM problem within the framework of IVNHFS. The process of calculation is simple and provides a new idea for solving decision-making problems under the environment of IVNHFS. In the end, a practical numerical example is given to verify the feasibility of the developed MADM method. As we know, similarity measure, correlation coefficient and entropy are important topics in fuzzy set theory. In the future, I will continue to study the similarity measure of neutrosophic HFSs

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