

# Inhomogeneous Navier-Stokes equations in the half-space with nonhomogeneous boundary values

Tongkeun, Chang<sup>2</sup> Bum Ja Jin<sup>2</sup>

1) *Department of Mathematics, Yonsei University*

2) *Department of Mathematics, Mokpo National University*

Corresponding Author : Bum Ja Jin, bumjajin@mokpo.ac.kr

## ABSTRACT

We are concerned with the global (in time) well-posedness issue for the initial boundary value problem pertaining to the inhomogeneous incompressible Navier-Stokes equations to a nonhomogeneous initial and boundary values:

$$\begin{aligned}D_t \rho + \operatorname{div}(\rho \mathbf{u}) &= 0, & \text{in } \mathbb{R}_+^n \times (0, T), \\D_t(\rho \mathbf{u}) + \operatorname{div}(\rho \mathbf{u} \otimes \mathbf{u}) - \Delta \mathbf{u} + \nabla p &= 0, & \text{in } \mathbb{R}_+^n \times (0, T), \\ \operatorname{div} \mathbf{u} &= 0, & \text{in } \mathbb{R}_+^n \times (0, T), \\ \mathbf{u} &= \mathbf{g}, & \text{on } x_n = 0, \\ (\mathbf{u}, \rho) &= (\mathbf{h}, \rho_0), & \text{on } t = 0, \\ \rho|_{\Gamma_{in}} &= \rho_\infty,\end{aligned}$$

where  $\rho$ ,  $\mathbf{u} = (u_1, \dots, u_n)$  and  $p$  stand for the density, velocity field and pressure of the fluid, respectively, depending on the time variable  $t \in \mathbb{R}_+$  and on the space variables  $x \in \mathbb{R}_+^n$ . Here  $\Gamma_{in}(t) = \{\mathbf{x}' \in \mathbb{R}^{n-1} | g_n(\mathbf{x}', t) < 0\}$ .

We show the global in time existence of a inhomogeneous fluid when the initial data is small perturbation of a homogeneous fluid.