THE LEAST-SQUARES FINITE ELEMENT METHOD WITH DUAL SYSTEM FOR HELMHOLTZ EQUATION

Nuree Song¹, Eunjung Lee¹

¹) School of Mathematics and Computing (Computational Science and Engineering), Yonsei University, Seoul 03722, KOREA

Corresponding Author: Eunjung Lee, eunjunglee@yonsei.ac.kr

ABSTRACT

This study employs the least-squares finite element method in a dual system to solve Helmholtz equation for large wavenumber k. The wavenumber, also called spatial frequency, causes oscillatory behaviors of the solution, making it difficult to resolve. In Helmholtz problem, the key is to control ‘pollution effect’, a property that is easily affected by the magnitude of k.

We define a model problem on a ring-shaped domain with Robin boundary condition which approximates the Sommerfeld radiation condition to ensure the unique solution. The Helmholtz equation is reconstructed into the first order system by introducing a new variable and scaling factors are inserted to the system to alleviate the dependency on k. $L^2$-adjoint for this system yields a dual problem and the least squares approach is applied to obtain a variational problem corresponding to residual minimization. By showing the unique existence of weak solution to this dual problem, we have the explicit forms of continuity and coercivity constants which depend on k. The a priori error estimation allows us to choose the proper scaling factors independent of k. Numerical experiments reveal that our approach effectively undermines the pollution effect.