EMERGENT DYNAMICS OF A THERMODYNAMIC CUCKER-SMALE ENSEMBLE ON COMPLETE RIEMANNIAN MANIFOLDS

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ABSTRACT

We study emergent collective behaviors of a thermodynamic Cucker-Smale (TCS) ensemble on complete smooth Riemannian manifolds. For this, we extend the TCS model on the Euclidean space to a complete smooth Riemannian manifold by adopting the work for a CS ensemble, and provide a sufficient framework to achieve velocity alignment and thermal equilibrium. As a concrete example, we study emergent dynamics of the TCS model on the unit \(d\)-sphere \(S^d\) in the proposed sufficient framework for a general setting. We show the asymptotic dynamics of the proposed TCS model on the unit \(d\)-sphere exhibits a dichotomy, either convergence to zero velocities or approach toward a common great circle. We also provide several numerical examples illustrating the aforementioned dichotomy on the asymptotic dynamics of the TCS particles on \(S^d\).