On minimal highly connected spanning subgraphs in dense digraphs

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ABSTRACT

In 1985, Mader [4] showed that every \( n(\geq 4k+3) \)-vertex strongly \( k \)-connected digraph contains a spanning strongly \( k \)-connected subgraph with at most \( 2kn - 2k^2 \) edges, and the only extremal digraph is a complete bipartite digraph \( DK_{k,n-k} \). Nevertheless, since the extremal graph is sparse, Bang-Jensen [1] asked whether there exists \( g(k) \) such that every strongly \( k \)-connected \( n \)-vertex tournament contains a spanning strongly \( k \)-connected subgraph with \( kn + g(k) \) edges, which is an “almost \( k \)-regular” subgraph.

Recently, the question of Bang-Jensen was answered [3] in the affirmative with \( g(k) = O(k^2 \log k) \), which is best possible up to logarithmic factor. In this talk, we discuss how to find minimal highly connected spanning subgraphs in dense digraphs as well as tournaments. In particular, we show [2] that every highly connected dense digraph contains a spanning highly connected subgraph that is almost \( k \)-regular, which yields \( g(k) = O(k^2) \) that is best possible for tournaments.

REFERENCES


