Global in time solvability of the Navier-Stokes equations in the half-space

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ABSTRACT

In this paper, we study the initial value problem of the Navier-Stokes equations in the half-space. Let a solenoidal initial velocity be given in the function space $\dot{B}^{\alpha-\frac{2}{2}}_{pq,0}(\mathbb{R}^n_+)$ for $\alpha+1=\frac{n}{p}+\frac{2}{q}$, $\frac{2}{q}<\frac{n}{p}+1$ and $0<\alpha<2$. We prove the global in time existence of weak solution $u\in L^q(0,\infty;\dot{H}^\alpha_p(\mathbb{R}^n_+))\cap L^{q_0}(0,\infty;L^{p_0}(\mathbb{R}^n_+))$ for some $p<p_0, 1<q_0<\infty$ with $\frac{n}{p_0}+\frac{2}{q_0}=1$, when the given initial velocity has small norm in function space $\dot{B}^{-\frac{2}{q_0}}_{p_0q_0,0}(\mathbb{R}^n_+)$ ($\supset \dot{B}^{\alpha-\frac{2}{2}}_{pq,0}(\mathbb{R}^n_+)$). The solution is unique in the class $L^{q_0}(0,\infty;L^{p_0}(\mathbb{R}^n_+))$. Pressure estimates are also given.