High order conservative Semi-Lagrangian scheme for the BGK model of the Boltzmann equation

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ABSTRACT

In this work, we present a conservative semi-Lagrangian finite-difference scheme for the BGK model. Classical semi-Lagrangian finite difference scheme do not necessarily conserve the total mass, density and energy, the error becoming more relevant as the Knudsen number is smaller and smaller [1]. To ensure conservation of the macroscopic variables, we propose a conservative correction procedure based on the flux difference form as in [2]. In general this procedure implies a stability restriction on the time step. When using classical continuous Maxwellian, conservation error is negligible if velocity space is resolved with sufficietnly large number of grid points. However, for a small grid in velocity space such error is not negligible. In order to overcome this issues and to ensure the machine precision conservation of mass, momentum and energy with a small number of velocity grid points, we use the discrete Maxwellian, introduced in [3]. In this way we can construct a semi-Lagrangian scheme which is Asymptotic Preserving for the underlying Euler limit, as the Knudsen number vanishes. The proposed conservative semi-Lagrangian scheme is applied to solve the BGK model and the effectiveness of the scheme is demonstrated by extensive numerical tests.

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