Stability and Hopf bifurcation Analysis in the Fractional order predator-prey biological economic system with Holling type II functional response

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ABSTRACT

Nowadays, fractional calculus has been considered as the novel mathematical model to describe various phenomena in nature. Due to the non-local property of the fractional derivative, the fractional-order differential equations are more suitable than integer-order ones in biological, economic ans social systems where memory effects are important. The predator-prey model is studied to describe the relationship between two species in biological systems in which one a predator feeds on the other prey.

In this work, we consider the fractional-order predator-prey biological economic model with Holling type II functional response,

$$\begin{aligned} D^{\alpha_1}x(t) &= x\left(d - kx - \frac{y}{a+x} - E\right), \\ D^{\alpha_2}y(t) &= y\left(-r + \frac{bx}{a+x}\right), \\ 0 &= E(px - c) - \upsilon. \end{aligned} \tag{1}$$

where $0 < \alpha_1, \alpha_2 \leq 1$. x and y represent the prey density and predator density at time t, respectively. d, r, a and b are prey intrinsic growth rate, predator mortality rate, half capturing saturation constant and maximal predator growth rate that are positive, respectively. $d/k > 0$ is the carrying capacity of the prey. $E(t)$ is the harvest effort. p denotes harvesting reward per unit harvesting effort for unit weight. c represents a harvesting cost per unit harvesting effort. υ is the economic profit.

We mainly discuss about the effect of economic profit υ and the fractional orders α_1 and α_2. We study stability analysis of this system (1) for both commensurate and incommensurate one and Hopf bifurcation analysis for both commensurate and incommensurate one. Furthermore, we give numerical simulations to support our analysis.

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