MULTIFIELD VARIATIONAL SECTIONAL ANALYSIS FOR ACCURATE STRESSES OF COMPOSITE BEAMS

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ABSTRACT

A multifield variational beam sectional analysis is proposed for anisotropic beams with arbitrary cross-sectional geometries and material distributions. Both stresses and three-dimensional (3D) warping deformations are treated as unknowns which are modeled using the isoparametric finite element (FE) shape functions. The nodal stresses are solved on each material domain resulting in the required continuity within the domain and discontinuity at the material interfaces. A generalized Timoshenko like stiffness matrix is constituted from the 3D warping solution inherently describing Timoshenko model for transverse shear, Poisson deformations, and associated couplings. The accuracy of the present analysis is demonstrated for benchmark composite beams with elastic couplings. The stress predictions from the present multifield formulation show excellent correlation with 3D FE results using fewer elements while maintaining the continuity requirements for multilayered composite beams as compared with the conventional displacement-based approach.

THEORY

The present formulation is developed for nonhomogeneous composite beams following the Reissner's multifield variational principle [1]. The 3D warping displacements and transverse sectional stresses (1 normal and 2 transverse components, called as reactive stresses as defined in [2]) are modeled as unknowns (field variables). The boundary effects due to nonuniform shear and torsional warping are incorporated which results in a 9×9 refined sectional stiffness matrix including classical and nonclassical elastic couplings.

The cross-sectional strain energy using Reissner's multifield variational principle [1] is given as

$$\delta U_s = \int_A [(\delta \varepsilon_n^a)^T \sigma_n^a + (\delta \varepsilon_s^a)^T \sigma_s^r + (\delta \sigma_s^r)^T (\varepsilon_s^a - \varepsilon_s^r)] dA \quad (1)$$

where superscripts *a* and *r* indicate active and reactive terms, and *s* and *n* represent the components on the section and normal to the sectional plane, respectively. The 3D warping displacements and reactive sectional stresses are approximated as linear functions of generalized stress resultants. Using the known warping and reactive stress coefficients, the generalized 6x6 Timoshenko stiffness matrix is derived.

NUERICAL RSULTS

The present analysis is validated for bending-torsion coupled composite blades originally studied by Chandra and Chopra [3], as shown in Fig. 1. The warping deformation modes are shown in Fig. 2 which indicate coupling between torsional (M_1) and bending (M_2) modes. Little extension-shear couplings can also be noted. The reactive stress coefficients computed exclusively using present analysis are also illustrated in Fig. 3.

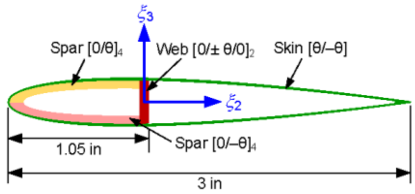
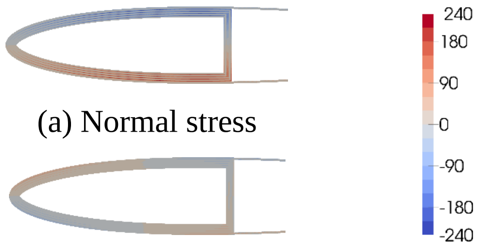


Figure 1. Schematic of composite blade

(a) Extension (F_1) (b) Shear (F_3) (c) Torsion (M_1) (d) Bending (M_2)

Figure 2. Selected warping deformations of blade



(a) Normal stress

(b) Transverse shear stress

Figure 3. Reactive stress coefficients of blade

Extensive validation study has been performed to validate the present beam theory developed based on multifield variational approach. These include experimental data [2], analytical shell-wall based mixed formulation [3], and displacement-based FE analysis RDSAC [4] respectively obtained for rectangular beams and thin-walled box-section beams with anisotropic couplings. The present values demonstrate excellent correlation with the experimental data other state-of-the-art composite beam analysis results.

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