An adaptive BDDC algorithm for three dimensional problems with an enhanced edge eigenvalue problem

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ABSTRACT

An adaptive BDDC (Balancing Domain Decomposition by Constraints) preconditioner is considered for three dimensional elliptic problems with coefficients of high contrast and random variation. For such problems, the classical primal unknowns in the BDDC preconditioner, such as values at subdomain vertices, and average values over subdomain edges or faces, are not enough to give a scalable coarse problem. Specially designed eigenvalue problems related to the estimate of condition numbers are considered and problematic eigenvectors are selected from the eigenvalue problems to enrich the set of primal unknowns so as to make the resulting coarse problem in the preconditioner more scalable. In three dimensions, eigenvalue problems are formed on each face or edge nodal equivalence classes. For the case of edges, proposed eigenvalue problems in previous studies are not satisfactory while those for faces perform very effectively. For the case of edges, the eigenvalues are not clearly separated in contrast to the case of faces and most eigenvectors are thus often selected in practice, see [1–3]. The eigenvalue problems can be enhanced by utilizing prior selected primal constraints as proposed in [4]. In this paper, this new idea is adopted when forming edge eigenvalue problems. The primal unknowns at subdomain vertices and the adaptive primal unknowns chosen from face eigenvalue problems are included when forming edge eigenvalue problems. For the BDDC preconditioner with the enhanced edge eigenvalue problems, the bound of condition numbers is shown to be dependent on the given tolerance value, that is used to select the problematic eigenvectors from face and edge eigenvalue problems, but independent of the contrast in the coefficients of the model problem. In addition, numerical results are provided to verify that the new edge eigenvalue problem gives a more optimal set of primal unknowns compared to those in the previous studies.

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