A Dynamical Systems Approach for the Shape Matching of Polytopes Along Rigid-Body Motions

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ABSTRACT

We present a dynamical systems approach for geometric matchings in an ensemble of polytopes along rigid-body motions. Each polytope can be characterized by a vertex set and edge or faces determined by vertices, and polygons and simplexes correspond to a polytope. For a geometric matching, we propose a system of dynamical system for the evolution of centroids and rotations of polytopes to match the vertices under rigid-body motions which can be decomposed as a composition of translation and rotations. Our proposed dynamical system acts on the product space $(\mathbb{R}^d \times SO(d))^N$. The evolution of centroids can be described by the coupled linear second-order dynamical system with diffusive linear couplings, whereas rotations for the matching of vertices are described by the Lohe matrix model on $SO(d)^N$. In particular, the Lohe matrix model has been derived from some set of physical principles compared to previous works in which the Lohe matrix model were employed as a system dynamics. This is a contrasted difference between earlier works on the Lohe matrix model which has been adopted a priori for an aggregate modeling of matrices. We also provide an analytical result leading to the complete shape matchings for an ensemble of congruent polytopes, and several numerical examples to illustrate analytical results visually.