

STRETCHED MOTZKIN PATHS AND THEIR APPLICATIONS

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ABSTRACT

Let $S = \{(a, 0), (b, 1), (c, -1)\}$ be a step set where a, b, c are integers such that $a \geq 0, b, c > 0$. In this paper we enumerate the lattice paths using the step set S bounded from below by the horizontal line $y = -\alpha$. In particular, for Motzkin paths, going down to $y = -1$ could be thought of as being on probation in academic circles or being in critical condition in medical terms. We examine four types of probationary conditions which lead to new interesting statistics on several well-known lattice paths. The enumeration allows us to prove the conjectures on the recurrence relations of several sequences posed by R. J. Mathar in OEIS.

REFERENCES

1. Krattenthaler, C., “Handbook of Enumerative Combinatorics”, Chapter 10, Lattice Path Enumeration, CRC Press, Boca Raton-London-New York, 2015.