Synchronization with exponentially decayed waiting time in a coupled integrable system

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ABSTRACT

In this talk, we explore the effect of coupling of two integrable systems where each system exhibits a periodic orbit. For two simple scenarios of couplings which are classified by the number of couplings, we found that two systems are eventually synchronized, and oscillatory behaviors of solutions in two patches become identical, i.e., the synchronized behavior emerges. In addition, we find that, whether single or complete couplings occur, the waiting time for the synchronization is exponentially decreasing as the coupling strength is intensified. We may apply our result to provide that species’ collective behavior according to migration between different patches may be easily predicted by the concept of a coupling of two systems, and additional information for waiting time and sensitivity of initial densities to yield synchronized behaviors.

INTRODUCTION

The issue for synchronization has been widely achieved in dynamical systems [1–3]. Such synchronization is usually associated to characteristics of couplings of nonlinear systems, and we apply the concept of couplings and synchronization to explore collective behavior in population systems.

The question how species biodiversity can be affected by flow of populations often becomes an important issue associated with long-range migration in ecological sciences. In particular, migration to different environments may occur by some or all populations, and affect biodiversity in their environments in either positive or negative way. In this regard, we wonder how species biodiversity can change if inter-patch migration occurs in groups of populations who stably coexist in each environment. To approach the issue, we consider the system of cyclically competing populations in two patches by assuming two specific scenarios: (a) only one species in each patch will migrate to the other patch each other, and (b) all species can migrate to the other patch. By means of the coupling of two systems with different initial conditions which can yield different oscillatory behaviors, we define the system in the macroscopic framework that the rate of inter-patch migration can be depicted by the coupling strength.

MAIN CONTRIBUTION

Under inter-patch migration of either single or all species movements between two patches, the synchronized coexistence can emerge by exhibiting stable periodic orbits. Two independent
periodic orbits with different oscillation amplitudes initially exhibit the collective behavior with the same motions on the same invariant space due to mutual inter-patch migration of populations. Computationally, for both cases of inter-patch migration with respect to the number of migrating species, we address the synchronization time, a waiting time to emerge synchronized coexistence, is exponentially decreasing as the strength of inter-patch migration increases. For the certain waiting time which is defined by the specific strength of inter-patch migration, in the case of single species inter-patch migration, we report the effect of initial conditions for the synchronization by investigating basin structures of initial conditions depending on the coupling strength [4]. With the given initial condition, a similar approach of considering effects of three different coupling parameters for the synchronization has been performed for the case of all species migration by employing basins of coupling parameters. In this case, we argue that the waiting time for the synchronization can differ according to the choice of coupling rates.

REFERENCES