Non-uniform Subdivision Schemes Reproducing Locally Different Curves

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ABSTRACT

The aim of this talk is to introduce a class of non-uniform subdivision schemes which reproduce locally different curves. In order to locally control the shape of the limit curve, we set a different shape parameter at each edge of the initial polygon. In each refinement step, the shape parameters are updated by the rule such that locally different analytic curves like trigonometric curves can be reproduced and blended. The regularity of the limit curves is discussed based on the asymptotically equivalent relation. We present some numerical examples in order to demonstrate the performance of our subdivision schemes.

PROPOSED SUBDIVISION SCHEMES

Let $\mathbf{f}^0 = \{f_i^0 : i \in \mathbb{Z}\}$ be a set of initial points. We assign bounded shape parameters $\mathbf{\Lambda}^0 = \{\lambda_i^0 \geq 0 : i \in \mathbb{Z}\}$ to the edges of the initial polygon. The proposed non-uniform subdivision scheme of order 2, denoted by S_2, defines a set of the new points $\mathbf{f}^{k+1}(= S_2\mathbf{f}^k)$ as

$$\begin{aligned} f_{2i}^{k+1} &= (1-\eta(\lambda_i^k))f_i^k + \eta(\lambda_i^k)f_{i+1}^k, \\ f_{2i+1}^{k+1} &= \eta(\lambda_i^k)f_i^k + (1-\eta(\lambda_i^k))f_{i+1}^k, \end{aligned}$$

where $\eta(x) := 1/(2(1+x))$. The shape parameters λ_i^k are updated by the rule $\lambda_{2i}^{k+1} = \lambda_{2i+1}^{k+1} = \sqrt{(1+\lambda_i^k)/2}$. For $d > 2$, let $\tau_d = \lceil \frac{d-2}{2} \rceil$, where $\lceil x \rceil$ indicates the smallest integer larger than x. The dth-order scheme S_d is defined as

$$(S_d\mathbf{f}^k)_i = \frac{1}{2^{d-2}} \sum_{j=0}^{d-2} \binom{d-2}{j} (S_2\mathbf{f}^k)_{i+j-\tau_d}, \quad d > 2.$$

REFERENCES

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