FAST TRANSITION LAYERS OF SLIGHTLY COMPRESSIBLE TWOPHASE FLOW

Hyeonseong Jin 1
1) Department of Mathematics, Jeju National University, Jeju, 63243, KOREA

Corresponding Author: Hyeonseong JIN, hjin@jejunu.ac.kr

ABSTRACT

We discuss the motion of weakly compressible twophase flow near initial time. There exist fast transition-layers in higher order of the inner limit process. In the transitional regions, we derive formal asymptotic expansions for the solutions of compressible equations. Specifically, under the singular limit process, we find the fast transitional variables in closed form.

The singular limit process of compressible twophase fluid flow is studied as the Mach number goes to zero. The incompressible limit of the compressible twophase flow equations is a time-singular and layer-type problem which requires advanced techniques in asymptotics [8]. The incompressible limit of the single phase compressible Euler or Navier-Stokes equations has been studied in higher space dimensions [1,6,9,10]. A uniformly valid asymptotic expansion describing a singular limit process exists uniquely. Each order of asymptotic expansions for the solutions of the compressible flow representing the incompressible limit process has an independent existence, defined as proportional to a derivative of the compressible solution with respect to \( \lambda \), the reciprocal of the Mach number, evaluated at the value \( \lambda \) of the expansion parameter. The uniformly valid outer limit asymptotic expansions have been derived [2,7] for the compressible twophase flow solutions describing the fluids away from the initial time. The slow variables with a slow scale of motion exist in the outer expansions and have been determined through second order in closed form. This is the order the incompressible pressure first appears. This paper is concentrated on the inner limit process of weakly compressible twophase flow describing the fluids near the initial time. The fast variables in the inner limit expansions contain fast scale acoustical oscillations on the fast time scale. Moreover, in higher order in \( \lambda^{-1} \), we are concerned here, more of fast transition-layers exist in the inner limit process. We derive the fast transition-layer expansions in the regions and determine the fast transitional variables which are used to derive uniformly valid inner limit asymptotic expansions by matching.

The compressible isentropic ideal twophase flow are nondimensionalized in the form of a nonlinear hyperbolic system

\[
\frac{\partial \beta_k^\lambda}{\partial t} + v_k^\lambda \frac{\partial \beta_k^\lambda}{\partial z} = 0,
\]

\[
\beta_k^\lambda \left( \frac{\partial \rho_k^\lambda}{\partial t} + v_k^\lambda \frac{\partial \rho_k^\lambda}{\partial z} \right) + \beta_k^\lambda \rho_k^\lambda \frac{\partial v_k^\lambda}{\partial z} + \rho_k^\lambda \left( v_k^\lambda - v^{**} \right) \frac{\partial \beta_k^\lambda}{\partial z} = 0,
\]

\[
\beta_k^\lambda \rho_k^\lambda \left( \frac{\partial v_k^\lambda}{\partial t} + v_k^\lambda \frac{\partial v_k^\lambda}{\partial z} \right) + \beta_k^\lambda \rho_k^\lambda \frac{\partial \rho_k^\lambda}{\partial z} + \lambda^2 \rho_k^\lambda \frac{\partial \rho_k^\lambda}{\partial z} + \lambda^2 \left( p_k^\lambda - p^{**} \right) \frac{\partial \beta_k^\lambda}{\partial z} = \beta_k^\lambda \rho_k^\lambda g(t),
\]

for the volume fraction \( \beta_k^\lambda \), velocity \( v_k^\lambda \), density \( \rho_k^\lambda \), and pressure \( p_k^\lambda \) of fluid \( k \), depending on a large dimensionless parameter \( \lambda \). Here \( p_k^\lambda = p_k^\lambda (\rho_k^\lambda) \) and an equation of state \( p_k^\lambda (\rho_k) = A_k \rho_k^{\gamma_k}, \gamma_k > 1 \) is given with \( (\partial p_k)/((\partial \rho_k) (\rho_k) > 0 \) for \( \rho_k > 0 \) and the entropy \( A_k \).
assumed to be constant within each fluid but $A_1 \neq A_2$. The fluids are distinguished by a subscript $k$, where $k = 1$ and $k = 2$ denote the light and heavy fluids, respectively.

As a background flow, the incompressible flow equations are considered for velocity $v_k^{\infty}$, volume fraction $\beta_k^{\infty}$, scalar pressure $p_k^{\infty}$ and constant density $\rho_k^{\infty}$ of the phase $k$. All state variables are assumed to be piecewise $C^1$ functions with discontinuous derivatives at the mixing zone edges $z = Z_k^{\infty}(t)$ of incompressible flow. Analytic solutions of the incompressible equations have been obtained in closed form [3-5].

One expects, under appropriate conditions, that the compressible multiphase flow solutions $\beta_k^\lambda$, $v_k^\lambda$, $p_k^\lambda$ converge to the incompressible solutions $\beta_k^{\infty}$, $v_k^{\infty}$, $p_k^{\infty}$ as $\lambda \to \infty$. The zero Mach limit of the compressible multiphase flow equations is a time-singular and layer-type problem which requires advanced techniques in asymptotics. In this paper we discuss the limiting behavior of the solutions $\mathcal{U}_k^\lambda \equiv (\beta_k^\lambda, v_k^\lambda, p_k^\lambda)^T$ of the compressible equations (1.1)-(1.3) near the initial time as $\lambda \to \infty$. We are concerned with the derivation of inner limit asymptotic expansions for the solutions of the compressible equations. In higher order in $\lambda^{-1}$, there exist fast transitional layers in the inner expansion similar to the transition regions in the outer asymptotic expansion. The first order inner expansion is defined by five regions, $\mathcal{E}_k^{(1)} \cup \mathcal{F}_k^{(1)} \cup M \cup \mathcal{F}_k^{(1)} \cup \hat{E}_k^{(1)}$, including two fast transition-layers through $z = \hat{Z}_i^{(1)}$, $i = k, k'$. See Fig. 1. We find the fast transitional variables of each order of $\lambda^{-1}$ in the fast transition-layers. They satisfy simple differential equations and they are solved in closed form.

![Figure 1: The five layers in the first order of the inner limit expansion](image-url)
References


