

Some inequalities concerning the maximum modulus of a polynomial

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Abstract

Let $P(z)$ be a polynomial of degree n not vanishing in $|z| < k$ where $k \geq 1$. It is known that

$$\begin{aligned} \text{Max}_{|z|=R>1}|P(z)| \leq & \frac{(R+k)^n}{(R+k)^n + (1+Rk)^n} \\ & \left\{ (R^n + 1)\text{Max}_{|z|=1}|P(z)| - \left(R^n - \left(\frac{1+Rk}{R+k} \right)^n \right) \text{Min}_{|z|=k}|P(z)| \right\}. \end{aligned}$$

In this paper, we obtain a refinement of this and many other related results.

Key words and Phrases: Polynomial, Zeros, Maximum modulus.

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