A new type smoothness indicator for the third order WENO schemes

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ABSTRACT

We present a new type smoothness indicator for the third order weighted essentially non-oscillatory (WENO) finite difference schemes for the conservation laws. WENO can provide the third convergence order in smooth regions. The proposed WENO scheme provides at least the same or improved behavior over the classical third-order WENO scheme, but its advantage seems more salient in one and two dimensional problems. Some numerical experiments are presented to demonstrate the performance of the proposed scheme.

INTRODUCTION

The goal of this paper is to introduce an improved version of the third-order WENO finite difference scheme for the approximation of hyperbolic conservation laws in the form

\[ q_t + f(q)_x = 0, \quad x \in \mathbb{R}, \quad t \geq 0, \]
\[ q(x, 0) = q_0(x), \quad (1) \]

with proper boundary conditions. Here, the function \( q = (q_1, \ldots, q_m) \) is an \( m \)-dimensional vector of conserved quantities and the flux \( f(q) \) is a vector-valued function of \( m \) components.