Finite element approximation of an incompressible chemically reacting non-Newtonian fluid

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ABSTRACT

We consider a system of nonlinear partial differential equations modelling the steady motion of an incompressible non-Newtonian fluid, which is chemically reacting. The governing system consists of a steady convection-diffusion equation for the concentration and the generalized steady Navier–Stokes equations, where the viscosity coefficient is a power-law type function of the shear-rate, and the coupling between the equations results from the concentration dependence of the power-law index. This system of nonlinear partial differential equations arises in mathematical models of the synovial fluid found in the cavities of moving joints. We construct a finite element approximation of the model and perform the mathematical analysis of the numerical method in the case of two and three space dimensions respectively. Key technical tools include discrete counterparts of the Bogovskiĭ operator, De Giorgi’s regularity theorem in two dimensions, and the Acerbi–Fusco Lipschitz truncation of Sobolev functions, in function spaces with variable integrability exponents.

REFERENCES