Optimal Preconditioners
on Solving the Poisson equation
with Neumann boundary conditions

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Abstract

We review Relaxed ILU and Perturbed MILU preconditioners in the case of Neumann boundary conditions, and present empirical results which indicate that the former is optimal in two dimensions and the latter is optimal in two and three dimensions. There is little difference in coding the optimal preconditioners and the conventional ones. A slight code change from the conventional to the optimal can result in significant speed ups in computational time. In our fluid simulation, the speed up reached about 3.3 and 2.0 with respect to Jacobi and ILU, respectively.

To the best of our knowledge, these empirical results have not been rigorously verified yet. We present a formal proof for the optimality of Relaxed ILU in rectangular domains, and discuss its possible extension to general smooth domains and Perturbed MILU.