Energy stable compact scheme for Cahn–Hilliard equation with periodic boundary condition

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ABSTRACT
We present a compact scheme to solve the Cahn–Hilliard equation with a periodic boundary condition, which is fourth-order accurate in space. We introduce schemes for two and three dimensions, which are derived from the one-dimensional compact stencil. The energy stability is completely proven for the proposed scheme based on the application of the compact method and well-known convex splitting methods. Detailed proofs of the mass conservation and unique solvability are also established. Numerical experiments are presented to demonstrate the accuracy and stability of the proposed methods.

INTRODUCTION

A phase-field model is a powerful mathematical tool for solving interfacial problems such as those involving solidification dynamics, multiphase fluid flows, vesicle membranes, and diblock copolymers. The Cahn–Hilliard (CH) equation, which was originally introduced to model the phase separation in a binary alloy [1], is typical phase-field models. The equation is

$$\frac{\partial \phi}{\partial t} = \Delta \mu, \quad \mu = F'(\phi) - \epsilon^2 \Delta \phi,$$

where $\mu$ is a chemical potential, $F(\phi)$ is the bulk energy density, and $\epsilon > 0$ is a coefficient related to the interfacial width. Here, we consider the rectangular domain $\Omega$ and the periodic boundary condition for both $\phi$ and $\mu$.

Many researchers have proposed various numerical methods to achieve phase-field models with energy stability and high-order accuracy. In this talk, we provide a simple compact scheme that is effective regardless of the dimensions under a periodic boundary condition and provide the proof of its energy stability.

NUMERICAL RESULTS

Simulations are performed with $\Delta t = 0.02/2^{10}$ until the final time $T_f = 0.02$. For the other parameters, $\epsilon = 0.02$, and the multigrid tolerance is $10^{-12}$.

Figure 1 (a) shows the relative $l_2$-errors with various grid sizes $N_x = N_y = 64, 96, 128, 192, 256$. In addition, Fig. 1(b) shows the errors with various grid sizes $N_x = N_y = N_z =$
Figure 1. Relative $l_2$-errors for CH equation.

32, 48, 64, 96, 128. The reference solutions for two dimensions and three dimensions are the numerical results with $N_x = 512$ and $N_x = 256$, respectively. Note that we set the reference solutions as the results from the compact scheme for both standard and compact cases. We use the cubic spline interpolation for the reference solution to define the error because $\Omega_h$ is a set of cell-centered grid points. Note that the error of the cubic interpolation bounds $O(h^4)$ for the function value. It is observed that the standard and compact schemes give second- and fourth-order accuracies in space, respectively.

REFERENCES