Computational Paradigms: Old and New

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ABSTRACT

Galerkin finite element methods based on simplicial/cubical meshes have been studied over the past half century and their mathematical analysis and computational engineering applications are well understood. Recently, polygonal/polytopal finite element methods have received considerable attention. This is because general meshes offer a very convenient framework for mesh generation, mesh deformations, fracture problems, composite materials, topology optimizations, mesh refinements and coarsening; for instance, to handle hanging nodes, different cell shapes within the same mesh and non-matching interfaces. Such a flexibility represents a powerful tool towards the efficient solution of problems with complex inclusions as in geophysical applications or posed on very complicated or possibly deformable geometries as encountered in basin and reservoir simulations, in fluid-structure interaction, crack propagation or contact problems.

In this talk, a new computational paradigm for discretizing PDEs is presented via staggered Galerkin method on general meshes. First, a class of locally conservative, lowest order staggered discontinuous Galerkin method on general quadrilateral/polygonal meshes for elliptic problems are proposed. The method can be flexibly applied to rough grids such as highly distorted meshes. Next, adaptive mesh refinement is an attractive tool for general meshes due to their flexibility and simplicity in handling hanging nodes. We derive a simple residual-type error estimator. Numerical results indicate that optimal convergence can be achieved for both the potential and vector variables, and the singularity can be well-captured by the proposed error estimator. Then, some applications to diffusion equations, Stokes equations, and linear elasticity equations are considered. Finally, we extend this approach to high-order polynomial approximations on general meshes. This is joint work with Dohyun Kim, Dong-wook Shin, and Lina Zhao.

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