Theory and practical applications of thin film flows

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The course will start with the standard Navier-Stokes equations and their reduction to the lubrication or thin film limit. Once the basic equations are found we will cover the following topics:

- **Flow in a closed channel** and comparison of the thin film limit with uni-directional flow.

  Applications: Any lubrication problem, e.g. lubricating gears, bearings, metered coating (such as blade coating), cold rolling of steel.

- **Free surface flows**. In the most difficult scenario, where surface tension drives the flow, this leads to a classical fourth-order nonlinear diffusion equation [1]. We will study various limits of this equation, e.g. flow driven by gravity and surface shear, and analytical solutions.

  Applications: Any coating problem, e.g. painting, manufacture of coated products, motion of rain down a window, condensate motion, lava flow etc.

- **Moving contact lines** and flow over a rough surface. When a film moves over a dry surface the bulk fluid catches up with the leading edge and starts to pile-up in a 'capillary ridge'. At the leading edge the theory predicts a stress singularity. This classical difficulty must be dealt with in order to model any thin film flow over a dry surface [2, 3].

- **Basic numerics**. In many practical situations it is not possible to solve the governing equations analytically. We shall explore some of the basic techniques required to solve the appropriate equations.

- **Spin coating and drop spreading**. We now move onto the the problem in a radial system. We will explore the axisymmetric spreading of a thin fluid layer and investigate the role of different forces such as surface tension, centrifugal and Coriolis.

  Applications: Spin coating is used in the manufacture of CDs, DVDs and computer disks, spin drying is a standard industrial process for fluid removal. Computer disks rely on the air bearing effect, namely the load bearing capacity of a thin rotating layer of air. Powder production, atomization and water flow on turbofan blades are also common examples, see [4, 5].

- **Thermal effects and phase changes**. Coatings often involve the coating fluid changing to a solid as it dries or freezes. This means that the flow equation must be coupled with (at least) one describing the solidification. We will consider freezing as the method of solidification and so introduce the appropriate heat equations in the solid and liquid and couple the equations through the Stefan condition, see [6, 7]. We will also show how the models apply to evaporation/condensation.

Applications: Painting, coating, ice accretion (in particular on aircraft and power lines).

- **Non-Newtonian fluids**. Until now the lectures will have dealt with Newtonian fluids, i.e. with a constant viscosity, see [8]. Many practical fluids have a viscosity that varies, for example with heat (such as oil) [9], shear stress (most polymer solutions) and pressure (oil, gases etc). We will investigate how these different fluid models affect the governing equations and solutions.

  Applications: Any coating/extrusion process involving polymers, e.g. manufacture of plastic sheeting, CD (etc) manufacture. Oil behaviour in gears (temperature effect), cold rolling of steel (pressure effect).

During the course we will also develop certain essential techniques. In particular we will use non-dimensionalization (to reduce equations to a simple form) and apply perturbation theory.

The student will require a basic knowledge of fluid and heat flow.

Bibliography

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Prof. Tim Myers

Professor Myers has many years of experience in both the modelling of thin films and their industrial applications. He has presented his work to industry, at numerous conferences around the world and written many papers on these topics, see [1, 2, 3, 4, 5], for example.

At Oxford University he worked in the Centre for Industrial and Applied Mathematics (OCIAM) as the Industrial Liaison Officer. There he worked on developing mathematical models for companies such as DuPont, Courtaulds and Pilkington.

After leaving Oxford he moved on to Cranfield University. There he worked on problems in flow over soil, water droplet motion through aircraft engines and the decommissioning of oil rigs. However, his main role was as the academic project manager in a consortium set up to develop a three-dimensional aircraft icing code. The models he developed for water flow and ice accretion during this time were incorporated into a commercial code, ICECREMO. This code is currently used by BAe Systems, Airbus UK, Rolls-Royce, Westland Helicopters, Dunlop Aerospace and Qinetiq, as part of their aircraft component validation process, see [6].

Prof. Myers currently works at the University of Cape Town, where he supervises 4 PhD students who are studying various aspects of thin film flow.

References

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