Consensus with switching topology and delay both in information and neighboring

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ABSTRACT

This article describes the effect of time delays on the consensus ability of systems when time delays are adjusted to both neighboring and state information. The modified protocols which time delays are applied also in neighboring can make distributed control more realistic and useful. We considered communication delays with directed switching topology in this paper.

PROBLEM DEFINITION

We consider single integrator MAS consisting of N agents with dynamics

$$\dot{x}_i(t) = \sum_{j=1}^{N} a_{ij}^{\sigma(t-\tau_{ij}(t))}[x_j(t-\tau_{ij}(t)) - x_i(t)] \tag{1}$$

for $i \in \mathcal{N} = \{1, 2, ..., N\}$, where $x_i(t) \in R^n$ is the state of agent i. The agents of the MAS are interconnected on a network in order to achieve consensus, i.e., $\lim_{t\to\infty} \|x_i(t) - x_j(t)\| = 0,$ for all $i, j \in \mathcal{N}$. Switching topologies are particularly interesting because they can be used to model important network properties like packet-loss or limited communication range in wireless networks. Communication delays are important to describe packet-delays, access delays, or propagation delays in communication networks.

The difference of this approach from that of the previous studies is that the neighboring $N_i(t)$ is also delayed with $\tau_{ij}(t)$ to $N_i(t - \tau_{ij}(t))$. Moreover, many real-world systems can be more realistically expressed by (1). Problems such as safety issues in distributed computing and bird flocking in biology have the property that the state information influences the neighboring ([1–3]). Therefore when the information is delayed the neighboring is delayed too. Also in sensor networks discrete communications with continuous computation which can be expressed by (1) has been modeled and analyzed [4]. Furthermore, (1) can be practically useful in the sense of memory optimization and energy efficiency.

The *switching topology* of the network is modeled using switching graphs. A switching graph $\mathcal{G}_{\sigma(t)} : R \to \mathfrak{G}$ is defined on a finite set $\mathfrak{G} = \{\mathcal{G}_p\}, p \in \mathcal{P} = \{1, 2, ..., P\}$, of P directed graphs $\mathcal{G}_p = (\mathcal{V}, \mathcal{E}_p)$ with identical node set $\mathcal{V} = \{1, 2, ...N\}$, each node corresponding to one

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agent, but different edge set $\mathcal{E}_p \subset \mathcal{V} \times \mathcal{V}$ and corresponding adjacency matrix $A^p = [a_{ij}^p] \in R^{N\times N}$, i.e., $a_{ij}^p > 0$ if $(j,i) \in \mathcal{E}_p$ and $a_{ij}^p = 0$ otherwise, where (j,i) represents a directed edge from vertex j to vertex i, i.e., information flows from agent j to agent i. We define the parent set $\mathcal{N}_i^p = \{j : (j,i) \in \mathcal{E}_p\}$ of node i and the largest and smallest entry of all adjacency matrices $\overline{a} = \max_{i,j,p} a_{ij}^p > 0$ and $\underline{a} = \min_{i,j,p:a_{ij}^p\neq 0} a_{ij}^p > 0$. We assume here that the graphs $\mathcal{G}_p$ do not have self-loops, i.e., $a_{ii}^p = 0$ for all i, p. The switching between the graphs is modeled by a function $\sigma : R \rightarrow \mathcal{P}$ that is piecewise constant from the right. We denote the time instances where σ swithes $t_\eta > t_{\eta-1}, \eta = 1, 2, ...$ with $t_0 = 0$. We assume there are infinitely many switching times and any two consecutive switching instants are separated by a dwell-time h_{DW}.

DELAYED CONSENSUS PROBLEM WITH DIRECTED TOPOLOGY

We define $\overline{\gamma_k}(t)$ and $\underline{\gamma_k}(t)$ as upper and lower bounds of the smallest hyper-rectangle which contains all $x_i(t+\eta)$ for $\eta \in [-\overline{\tau}, 0]$ with dimension k, respectively. The proof directly follows from the previous research presented in [5] and are omitted.

Assumption 1 $A^{\sigma(t)}$ *is uniformly quasi-strongly connected, that is there exists* $\mathfrak{T} > 0$ *s.t. the union graph* $(\mathcal{V}, \bigcup_{\tau\in[t,t+\mathfrak{T}]} \mathcal{E}_\tau)$ *contains a spanning tree for all* $t \geq 0$.

Lemma 1 $\overline{\gamma_k}(t)$ *is non-increasing and* $\underline{\gamma_k}(t)$ *is non-decreasing with* (1) *under Assumption 1.*

Lemma 2 *If there exist* $t_0, k, \alpha_1, I_1 \subseteq \{1, 2, \cdots, N\}$ *s.t. for all* $i \in I_1$ $x_{i[k]}(t_0) \leq \overline{\gamma_k}(t_0) - \alpha_1$.

Then it satisfies that $x_{i[k]}(t_0 + 2\overline{\tau} + \mathfrak{T} + \eta) \leq \overline{\gamma_k}(t_0) - \beta_1$ *for all* $\eta \in [-(2\overline{\tau} + \mathfrak{T}), 0]$ *with* (1) *under Assumption 1. (*$\beta_1 = e^{-\overline{a}N(2\overline{\tau}+\mathfrak{T})}\alpha_1$*)*

Lemma 3 *If there exist* $t_0, k, \beta_1, I_1 \subseteq \{1, 2, \cdots, N\}$ *s.t. for* $\forall i \in I_1, \forall \eta \in [-(2\overline{\tau} + \mathfrak{T}), 0]$ *it satisfies* $x_{i[k]}(t_0 + 2\overline{\tau} + \mathfrak{T} + \eta) \leq \overline{\gamma_k}(t_0) - \beta_1$ *and there exists* i_{R_1} *in* I_1 *which is a root of* $\int_{t_0}^{t_0+\mathfrak{T}} \mathcal{A}^{\sigma(s)} ds$.

Then it satisfies $x_{I[k]}(t_0 + 2\overline{\tau} + \mathfrak{T}) \leq \overline{\gamma_k}(t_0) - \alpha_2$ *for* $\exists I \in \{1, 2, \cdots, N\} - I_1$ *with* (1) *under Assumption 1. (*$\alpha_2 = e^{-\overline{a}N[\overline{\tau}+\mathfrak{T}]}(1 - e^{-(\underline{a}+\overline{a}N)\frac{\epsilon}{2\overline{a}}})\frac{\underline{a}}{\underline{a}+\overline{a}N}\beta_1$*)*

Theorem 1 (1) *achieves consensus under Assumption 1.*

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