

A Non-Overlapping Domain Decomposition Method for a Discontinuous Galerkin Method

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ABSTRACT

In this talk we will discuss a non-overlapping domain decomposition preconditioner for a symmetric interior penalty Galerkin (SIPG) method for the heterogeneous elliptic problem. The proposed preconditioner is based on balancing domain decomposition by constraints (BDDC) methodology. Theoretical results on the condition number estimate of the preconditioned system will be presented along with numerical results.

The linear system resulting from the SIPG discretization is ill-conditioned with respect to the discontinuous coefficient ρ in the heterogeneous elliptic problem as well as mesh refinements. In the design of our preconditioner, the discontinuity of finite element functions causes the major difficulty in localizing the bilinear form and deriving a key norm estimate. Firstly, a subspace decomposition is introduced for the localization which is crucial in constructing a BDDC preconditioner. Secondly, an enriching technique of the discontinuous finite element space plays a major role in deriving the norm equivalence which is crucial for condition number estimates of preconditioned Schur complement systems.

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