

ASYMPTOTIC ANALYSIS OF FIRST EXIT TIME OF A LÉVY FLIGHT

Yooru KIM¹, Irem KOPRULU² and Ness B. SHROFF²

1) *Department of Mathematics, University of Ulsan, Ulsan 680-749, KOREA*

2) *Department of Electrical and Computer Engineering, The Ohio State University, Columbus, OH 43210, USA*

Corresponding Author : Yooru KIM, yrkim@ulsan.ac.kr

ABSTRACT

A Lévy flight is a class of random walks that is characterized by a heavy-tailed step-length distribution. In this talk, we discuss the first exit time of a Lévy flight from a bounded region, in particular a closed 1-ball or 2-ball.

For one-dimensional Lévy flights, the authors in [1,2] have studied the average first exit time from a bounded interval. A later study [3], however, points out that the analytical results in [1,2] are questionable due to inappropriate non-local boundary conditions. The authors of [3] provide a numerical study to verify their results, and the analytic solution is left open. For two-dimensional Lévy flights, there is only numerical study on the mean first exit time [4], and there are no analytic results for either the distribution or moments of the first exit time.

In this talk, we present a unifying framework that enables us to analyze the distribution and the mean of the first exit time for both the one- and two-dimensional Lévy flights. Using the framework, we characterize tight upper and lower bounds on the tail distribution of the first exit time, and identify the exact asymptotics of the mean first exit time for a given range of step-length distribution parameters.

REFERENCES

1. Buldyrev, S. V., Havlin, S., Kazakov, A. Y., da Luz, M. G. E., Raposo, E. P., Stanley, H. E. and Viswanathan, G. M., “Average time spent by Lévy flights and walks on an interval with absorbing boundaries”, *Phys. Rev. E*, Vol. 64, 2001, 041108.
2. Buldyrev, S. V., Gitterman, M., Havlin, S., Kazakov, A. Y., da Luz, M. G. E., Raposo, E. P., Stanley, H. E. and Viswanathan, G. M., “Properties of Lévy flights on an interval with absorbing boundaries”, *Physica A*, Vol. 302, 2001, pp. 148–161.
3. Dybiec, B., Gudowska-Nowak, E. and Hänggi, P., “Lévy-Brownian motion on finite intervals: mean first passage time analysis”, *Phys. Rev. E*, Vol. 73, 2006, 046104.
4. Vahabi, M., Schulz, J. H. P., Shokri, B. and Metzler, R., “Area coverage of radial Lévy flights with periodic boundary conditions”, *Phys. Rev. E*, Vol. 87, 2013, 042136.