

Asymptotic Analysis of Narrow Escape Problem in Domain with Long Neck

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ABSTRACT

The narrow escape problem arises in deriving the asymptotic expansion of the solution of an inhomogeneous mixed Dirichlet-Neumann boundary value problem. In this paper, we mainly deal with narrow escape problem in a smooth domain connected to a long neck-Dendritic spine shape domain, which has a certain significance in biology. Since the special geometry of dendritic spine, we develop a new model for solving this narrow escape problem which is Neumann-Robin Boundary Model. This model transform spine singular domain to smooth spine head domain by inserting Robin boundary condition to the connection part between spine head and neck. We rigorously find the high-order asymptotic expansion of Neumann-Robin Boundary Model and apply it to the solution of narrow escape problem in a dendritic spine shape domain. Our results show that the asymptotic expansion of the Neumann-Robin Boundary Model can be easily applied to the narrow escape problem for any smooth spine head domain with straight spine neck. By numerical simulations, we show that there is great agreement between the results of our Neumann-Robin Boundary Model and the original escape problem. In this paper, we also get some results for non-straight long spine neck case by considering curvature of spine neck. In this lecture, three dimensional case is also considered.

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