

ADAPTIVE GRID SIMULATION OF HYPERBOLIC PDES

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ABSTRACT

In this talk, we present an adaptive grid method for hyperbolic conservation laws. A multiresolution analysis, based on a biorthogonal family of interpolating scaling functions and lifted interpolating wavelets, is used to dynamically adapt grid points according to the physical field profile in each time step. Traditional finite-difference schemes with fixed stencils produce high oscillations around discontinuities. We combine high-resolution schemes, which are suitable for capturing singularities, and apply a finite-difference approach to the scaling functions at other points. We use a total variation diminishing Runge-Kutta method for the time update. The computational cost is proportional to the number of points present after compression. We provide several numerical examples to verify our approach.

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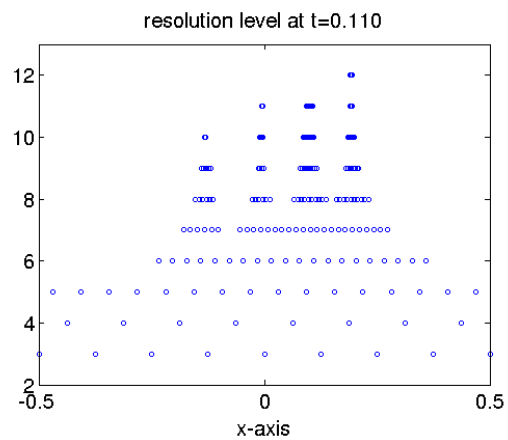
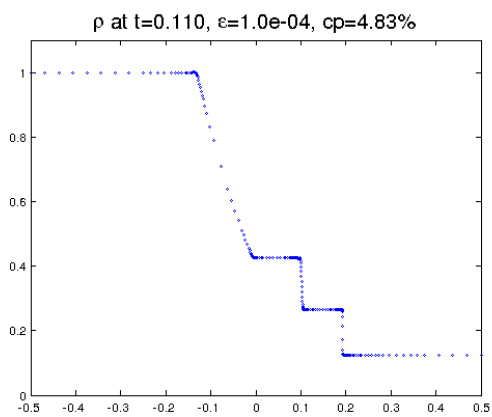
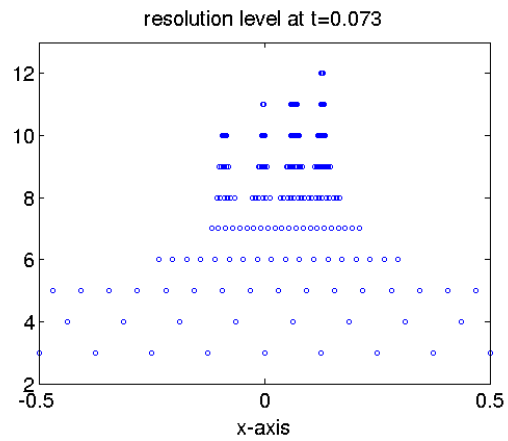
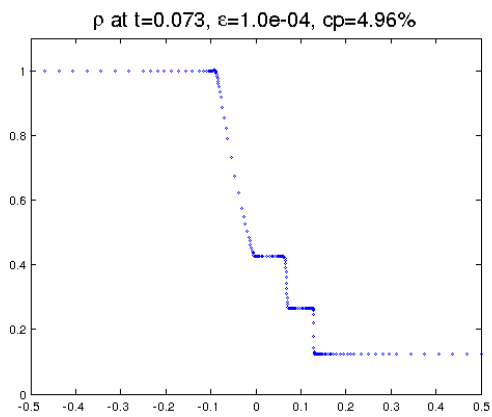
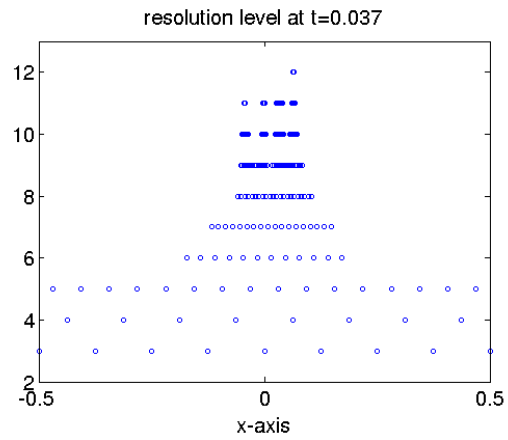
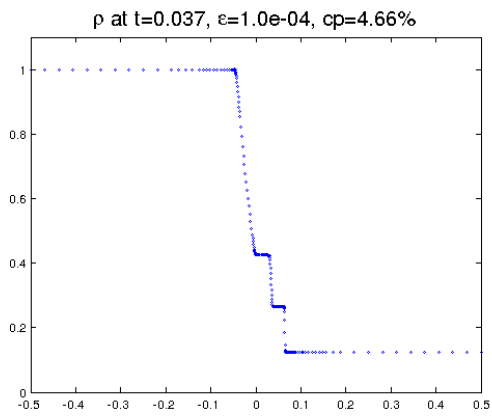


Figure 1. Evolution of density in the Sod shock-tube problem