Mathematical Model and Stability Analysis of Hepatitis B

Kihoon Jang [1] and Hee-Dae Kwon [1]

1) *Department of Mathematics, Inha University, Incheon 402-751, Korea*

Corresponding Author : Kihoon Jang, cdma1988@naver.com

ABSTRACT

Chronic hepatitis B virus (HBV) infection is one of serious health problems in the world. In this talk, we study HBV viral dynamics using a modified model of viral infection introduced by Neumann et al [3]. Our model contains four differential equations describing the change in the number of uninfected or target cells (T), infected cells (I), viral load (V), and immune effectors (E) with both cytolytic and noncytolytic activities. The mathematical model is as follows:

$$\begin{aligned}
\frac{dT}{dt} &= S - d_T T - (1-\eta) bVT + \alpha f IE \\
\frac{dI}{dt} &= (1-\eta) bVT + mI - d_I I - \alpha IE \\
\frac{dV}{dt} &= (1-\epsilon) pI - cV \\
\frac{dE}{dt} &= S_E + \frac{B_E IE}{(I + K_E)} - D_E E.
\end{aligned}$$

Using the next generation approach [2], we derive a threshold quantity (basic reproduction number, R_0) which is critical to the asymptotic stability of the virus-free equilibrium point. We also show that there exists a unique the chronic equilibrium point when the basic reproduction number is greater than one. Moreover, we investigate the relationship between the basic reproduction number and treatment efficacy of inhibiting de novo infection and viral production.

REFERENCES

1. Hwi Young Kim, Hee-Dae Kwon, Tae Soo Jang, Jisun Lim, Hyo-Suk Lee, Mathematical Modeling of Triphasic Viral Dynamics in Patients with HBeAg-Positive Chronic Hepatitis B Showing Response to 24-week Clevudine Therapy, *PLoS ONE* (2012) 7(11): e50377
2. L.J.S. Allen, *Mathematical Epidemiology*
3. Neumann AU, Lam NP, Dahari H, Gretch DR, Wiley TE, et al., Hepatitis C viral dynamics in vivo and the antiviral efficacy of interferon-alpha therapy, *Science* (1998) 282: 103-107.