

STRESS CONCENTRATION BETWEEN CLOSELY LOCATED STIFF INCLUSIONS

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ABSTRACT

If stiff inclusions are closely located, then stress, which is the gradient of the solution, may blow up as two inclusions approach to each other. In this work we derive optimal asymptotic formula describing the behavior of the high stress concentration in the narrow region between two inclusions. We first show that the solution for the case when two inclusions are touching each other decays exponentially near the touching point. We also prove a similar fact when two inclusions are closely located and we then use this fact to show that the stress concentration factor, which is a magnitude of the stress concentration, tends to a certain integral of the solution to the touching case.

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