# Regularity Criterion for Non-Newtonian fluids with No-slip Boundary Condition

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## ABSTRACT

We consider unsteady incompressible fluids in a smooth bounded domain $\Omega \subset \mathbb{R}^n$, $n = 2, 3$, which are described by the system of equations

$$\mathbf{u}_t + (\mathbf{u} \cdot \nabla)\mathbf{u} - \nabla \cdot T + \nabla p = \mathbf{0}, \qquad \nabla \cdot \mathbf{u} = 0. \tag{1}$$

Here, $u_t = \frac{\partial u}{\partial t}, \nabla = (\frac{\partial}{\partial x_1}, \cdots, \frac{\partial}{\partial x_n})$. The initial condition is prescribed by $\mathbf{u}|_{t=0} = \mathbf{a}$ and the Dirichlet boundary condition is prescribed by $\mathbf{u}|_{\partial\Omega} = 0$, where $\mathbf{u} = (u_1, \cdots, u_n)$ is the velocity, $p$ is the pressure, $T$ is the extra stress tensor, and $\mathbf{a}$ is the initial velocity of the fluid. We consider a constitutive relations for $T$ of the form $T = T(D)$, where $D$ denotes the symmetric part of the velocity gradient and $|D|$ denotes the usual Euclidean matrix-norm, that is, $D = (D_{ij})_{i,j=1,\cdots,n}$ and $|D|^2 = \sum_{i,j=1}^n D_{ij}^2$ for $D_{ij} = D_{ij}(\mathbf{u}) = \frac{1}{2}(\frac{\partial u_j}{\partial x_i} + \frac{\partial u_i}{\partial x_j})_{i,j=1,\cdots,n}$. In this work, we consider the type

$$T = (1 + |D|^2)^{\frac{r}{2}} D. \tag{2}$$

The existence of weak solutions of the equations (1) with Dirichlet boundary conditions for $r \geq \frac{n-2}{n+2}$ had first appeared in [13], which is unique for $r > \frac{n-2}{2}$. The existence of a weak solution with the Dirichlet boundary condition is extended to $1 > r \geq 0$ in [15], where the solution is strong for $1 > r \geq \frac{1}{4}, n = 3$, (see also [7], [17]). In [6] a regularity is shown for $r > \frac{n-2}{n+2}$. The existence result with the Dirichlet boundary condition has been extended to the case $r > -\frac{4}{n+2}$ in [12].

We are interested in the regularity properties of the weak solutions when the constitutive relations (2) has been considered. The existence of a local in time strong solution with space periodic boundary is known for $r > -\frac{1}{3}$ in [14], for $0 > r > -\frac{3}{5}$ in [11], and for $0 > r > -1$ in [8]. In [8], the result is for the model $T = |D|^r D$. In [9] the existence of a local in time strong solution for any $r > -1$ with no-slip or slip boundary conditions (or global time existence for small data) is shown. Recently, in [3] for the model $T = |D|^r D$ instead of (2) regularities are obtained; global in time for $r \geq \frac{n-2}{n+2}$, and local in time for $0 > r > \frac{n-4}{n+2}$.

In this work, we find Serrin type regularity criteria for the vorticity, and for the velocity. Serrin regularity criteria for the Navier-Stokes equations ($r = 0$) have been studied by many researchers [18]. Later, the two component regularity criterion was studied in [10] for the vorticity. The two component regularity for the velocity was done in [1] in 1997, which was publishded in [5], and the one component regularity criterion in [16]. For non-Newtonian fluids with the periodic boundary condition, such a regularity criterion is studied in [2] for shear thinning fluid, in [4] for shear thickening cases.

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