Surface reconstruction from parallel contours with exact contour constraints

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ABSTRACT

Images obtained from MRI or CT scanner are two-dimensional and parallel. In some cases, we need to reconstruct a surface from contours on 2D images. For example, currently Magnetic Resonance Electrical Impedance Tomography (MREIT) recovers conductivity in the human body by solving two-dimensional problems. However, for more accurate conductivity reconstruction, we need to solve three-dimensional forward problems and create a surface as computational domain. In addition, 3D reconstruction of the blood vessel or the organ is necessary in computational biology.

We consider two requirements for the surface reconstruction. First, it is required that the reconstructed surface satisfies exact contour constraints, which means that the surface exactly passes through each given contour. Secondly, the surface should be smooth enough because blood vessels or organs of the human body are smooth. To our best knowledge, there are no methods to satisfy the two requirements in present. For instance, minimizing the distance from a point-cloud set on the contours to the surface is not sure to satisfy the exact contour constraints. To solve PDEs in neighboring contours also has a problem because we don't know the boundary conditions.

Here, we propose a method to satisfy the two requirements. After defining an energy of the surface as the gradient of the normal vector, we minimize the energy. Then, the surface becomes to satisfy the smoothness requirement. For exact contour constraints, we express the surface using a level set function, and assign values of level set function on the planes containing the contour. Therefore we get an energy minimization problem with constraints. However, the Euler-Lagrange equation for this minimization problem is a fourth order PDE. To reduce the order of the PDE, we introduce an auxiliary variable of the normal vector, then the Euler-Lagrange equation becomes a 2nd order PDE. The minimization problem with constraints can be solved using the augmented Lagrangian method, and the solution is the surface which we look for. Implementation of the algorithm and numerical experiments are presented.

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