Design of an observer for linear hybrid dynamical systems with the information of jump times

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ABSTRACT

The purpose of this paper is to discuss a problem regarding the design of observers for linear hybrid systems, which exhibit characteristics of both continuous-time linear systems and discrete-time linear systems. We propose an observer which consists of the developed observers for continuous-time and discrete-time linear systems, when jumps of the state can be detected from the output measurement of hybrid systems. The proposed observer guarantees that the error of the state estimation converges to zero exponentially and we provide constructive conditions for the observer design in terms of Linear Matrix Inequalities (LMIs).

LINEAR HYBRID SYSTEMS

The paper uses the hybrid system framework summarized in [1]. Consider a linear hybrid dynamical system with the output y, given by

$$\begin{aligned} \dot{x} &= Ax + Bu, && x \in C \\ x^+ &= Fx, && x \in D \\ y &= Hx, && x \in \mathbb{R}^n, u \in \mathbb{R}^p \end{aligned} \tag{1}$$

with the initial condition x_0 and the input u, where A, B, H, and F are constant matrices of suitable sizes. The continuous dynamics of the system (1) may occur when the state x belongs to the flow set $C \subset \mathbb{R}^n$ and the discrete dynamics of the system (1) may occur when the state x belongs to the jump set $D \subset \mathbb{R}^n$. We suppose that C and D are closed sets and all solutions to (1) are complete. And assume that jumps of the state x can be detected from the output measurement. Then, adding a discrete state variable $q \in \mathbb{R}$ to the system, which informs the jumps of x before the discrete events occur, the linear hybrid system can be rewritten as

$$\begin{bmatrix} \dot{x} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} Ax + Bu \\ 0 \end{bmatrix}, \qquad (x, q) \in C \times \{0\}$$

$$\begin{bmatrix} x^+ \\ q^+ \end{bmatrix} = \begin{bmatrix} x \\ 1 - q \end{bmatrix}, \qquad (x, q) \in D \times \{0\}$$

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$$\begin{bmatrix} x^+ \\ q^+ \end{bmatrix} = \begin{bmatrix} Fx \\ 1-q \end{bmatrix}, \qquad (x,q) \in D \times \{1\}$$
$$y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} Hx \\ q \end{bmatrix}, \tag{2}$$

with the initial condition $(x(0,0), q(0,0)) = (x_0, 0)$. y is a new output including the information of jump times. In (2), q jumps from 0 to 1 before x jumps to Fx according to the discrete dynamics when $x \in D$. And q comes back to 0 from 1 when x jumps to Fx. Therefore, q can be considered as the part of the system output to inform the time instants when the discrete events of x occur, which gives the main idea to solve the observer design problem for linear hybrid systems.

OBSERVER DESIGN PROBLEM

One of observers to estimate x in (2) is the hybrid dynamical system of the form

$$\begin{aligned} \dot{\hat{x}} &= A\hat{x} - L_c H\hat{x} + L_c y_1 + Bu, \qquad & (\hat{x}, y_2) \in \mathbb{R}^n \times \{0\} \\ \hat{x}^+ &= F\hat{x} - L_d H\hat{x} + L_d y_1, \qquad & (\hat{x}, y_2) \in \mathbb{R}^n \times \{1\} \end{aligned} \tag{3}$$

if there exists $P = P^T > 0$, L_c, and L_d such that

$$\begin{aligned} P(A - L_c H) + (A - L_c H)^T P < 0, \\ (F - L_d H)^T P (F - L_d H) - P < 0. \end{aligned} \tag{4}$$

The proposed observer (3) simply consists of the continuous-time and discrete-time Luenberger observer. The observer (3) guarantees that the state estimation error $e(t,j) := \hat{x}(t,j) - x(t,j)$ converges to zero exponentially, which means that there exist $\lambda > 0$ and $\gamma \geq 1$ such that $|e(t,j)| \leq \gamma e^{-\lambda(t+j)} |e(0,0)|$ for all $(t,j) \in$ dom X, where $X = [\hat{x}^T x^T q]^T$. (t,j) is a hybrid time where $t \in \mathbb{R}_{\geq 0}$ is time and $j \in \mathbb{N}$ is the number of jumps and dom X is the hybrid time domain of the hybrid arc X.

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