# On the boundary integral equations for the ideal jet flows

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**ABSTRACT**

The inviscid, incompressible, and irrotational flow with sufficiently high speed passes through the air is called the jet (or cavity). Despite the fact that the governing equation for the jet flow is just the Laplace equation, it is hard to solve since this problem involves free boundaries to be determined as a major part of solution. The hodograph mapping is one of conventional approaches to solve this problem but it is still available and being studied for a restricted type of nozzle shapes[1]. In the early 1980's, the rigorous theories for jets appears in which the variational principle with parameters is used[2]. This variational approach enables us to use the finite element formulation. Here, we propose an effective formulation using the boundary integral equations with appropriate layer potentials in order to solve the impinging jets with arbitrary shape of nozzles.

## PROBLEM STATEMENT

The ideal jets in a two-dimensional domain $\Omega$ can be described with a stream function $\psi(\mathbf{x})$ since it is incompressible. In addition, it is assumed that the jet is issuing from an arbitrary nozzle. We place the nozzle on the left half plane ($x < -a$). Particularly, we assume that one part of the nozzle is composed of two straight lines parallel to each other as illustrated in Fig. 1. The jet issuing from the nozzle impinges on the vertical wall ($x = 0$) and it flows separately along the wall.

Physically, the stream function $\psi(\mathbf{x})$ is a harmonic function defined only on the jet region bounded by the nozzle, the free boundaries, and the wall. The boundary conditions for $\psi$ comes from the no penetration condition on the wall as well as on the free boundary and as a result it has a constant value on each connected boundaries. In calculating the stream function to satisfy these boundary conditions, there happens an obstacle that we do not know the free boundaries in advance.

Of more importance is that the stream function meets the constant speed condition on the free boundaries which is represented by means of the Neumann boundary condition on them. Therefore, we have two types of boundary conditions on the free boundaries, called the Dirichlet and Neumann boundary conditions. To the second order elliptic problems, only either one boundary condition is needed for the uniqueness in general. The additional boundary condition actually enables us to determine the free boundary.

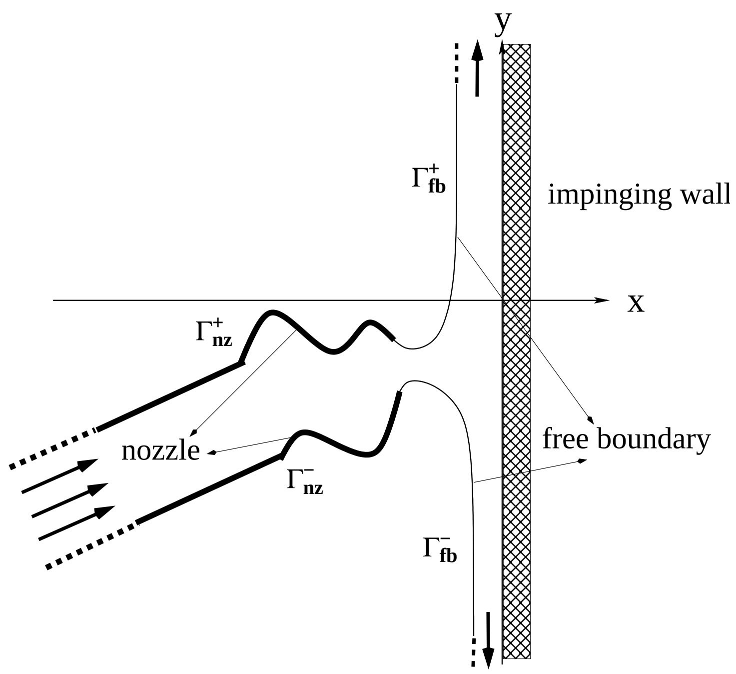


Figure 1. Configuration of the impinging jet problem

## FORMULATION AND RESULTS

In fact, there exist unknown parameters, the speed on the free boundary and the stream function value on the impinging wall. Here, it is what we only know that the speed on the free boundary is constant. It is of worth to remark that the latter value is zero if the flow is symmetric in case of the symmetric nozzle. However, in general, the latter value on the wall determines the ratio of jet flows separated in two directions after impinging.

The formulation is based on the single layer potentials distributed on the nozzle and free boundaries using a modified kernel to get rid of the layer potential on the impinging wall boundary. It can be done by the method of mirror image which is well known.

Finally, by using the potential theories for single and double layer potentials and by proving the boundedness of the potential functions, we obtain the following integral equations,

$$\begin{aligned}
&\pm Q - c + \lambda\left(\int_{\Gamma^+_{fb}} K(\mathbf{x},\mathbf{y})d\Gamma_{\mathbf{y}} - \int_{\Gamma^-_{fb}} K(\mathbf{x},\mathbf{y})d\Gamma_{\mathbf{y}}\right)\\
&\quad = \int_{\Gamma^+_{nz}} \mu^+_{nz}(\mathbf{y})K(\mathbf{x},\mathbf{y})d\Gamma_{\mathbf{y}} + \int_{\Gamma^-_{nz}} \mu^-_{nz}(\mathbf{y})K(\mathbf{x},\mathbf{y})d\Gamma_{\mathbf{y}}, \qquad \mathbf{x}\in\Gamma^{\pm}_{nz}\cup\Gamma^{\pm}_{fb},\\
&\lambda\left(\pm\frac{1}{2} + \int_{\Gamma^+_{fb}} \frac{\partial}{\partial \mathbf{n_x}}K(\mathbf{x},\mathbf{y})d\Gamma_{\mathbf{y}} - \int_{\Gamma^-_{fb}} \frac{\partial}{\partial \mathbf{n_x}}K(\mathbf{x},\mathbf{y})d\Gamma_{\mathbf{y}}\right)\\
&\quad = \int_{\Gamma^+_{nz}} \mu^+_{nz}(\mathbf{y})\frac{\partial}{\partial \mathbf{n_x}}K(\mathbf{x},\mathbf{y})d\Gamma_{\mathbf{y}} + \int_{\Gamma^-_{nz}} \mu^-_{nz}(\mathbf{y})\frac{\partial}{\partial \mathbf{n_x}}K(\mathbf{x},\mathbf{y})d\Gamma_{\mathbf{y}}, \qquad \mathbf{x}\in\Gamma^{\pm}_{fb},
\end{aligned}$$

where $\pm Q$ are the flux conditions enforced on the nozzle boundaries, $\Gamma^{\pm}_{nz}$ as well as on the free boundaries $\Gamma^{\pm}_{fb}$, $\lambda$ is the constant speed on the free boundaries, and $c$ is the stream function value on the impinging wall.

The point is to find a way to calculate the solutions, $\Gamma^{\pm}_{fb}$, $\mu^{\pm}_{nz}$, $\lambda$, and $c$, from the highly non-linear integral equations in the above. The numerical method based on an efficient iterative algorithm, including the numerical results, is presented at another talk in this conference.

## REFERENCES


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2. Alt, H.W., Caffarelli, L.A. and Friedman, A. , “Asymmetric Jet Flows”, *Communications on Pure and Applied Mathematics*, Vol. 35, 1982, pp. 29-68.