A DISCONTINUOUS GALERKIN METHOD WITH LAGRANGE MULTIPLIER FOR HYPERBOLIC CONSERVATION LAWS

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ABSTRACT

We develop a discontinuous Galerkin method with Lagrange multiplier (DGLM) to approximate the solution to the hyperbolic conservation laws with boundary conditions. Lagrange multipliers are introduced on the edge/face of the element via weak divergence. A Dirichlet boundary condition is weakly imposed on the edge/face of the element. Normal fluxes are shown to be continuous across the edge/face of the element. The final global system has substantially reduced numbers of unknowns of the standard DG methods. Neither numerical fluxes from finite volume/difference method nor additional shock capturing/artificial dissipation terms are considered. For the time discretization, backward Euler difference method is used. Stability of the approximate solution is proved in energy norm. Discontinuity of the solution is allowed in the error analysis. Local error estimates of $\mathcal{O}(h^{r+\frac{1}{2}} + \Delta t)$ with $P_r(E)$ elements $(r \geq \frac{d+1}{2})$ are derived, where h and Δt are the maximum diameter of the elements and time steps, respectively, and d is the dimension of the spatial domain. The high order approximation is obtained under an appropriate condition on the stabilizing parameter. It is shown that the method preserves the property of the local mass conservation. An explanation on algorithmic aspects is given. Some computational issues are addressed. An extension of the DGLM to an elliptic problem will be also addressed.