The impact of heterogeneous mixing on the transmission dynamics for a two-group influenza model

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ABSTRACT

A two-group influenza model is considered to study the impact of heterogeneous mixing on the transmission dynamics. We investigate the role of heterogeneous mixing on the incidence and the final epidemic size in both group. In particular, the results are compared under distinct mixing scenarios such as homogeneous mixing, proportionate mixing and arbitrary mixing.

A TWO-GROUP INFLUENZA MODEL WITH HETEROGENEOUS MIXING

We present the simple SIR epidemic model by allowing the possibility of subgroups with different activity levels and heterogeneous mixing between subgroups. This possibility can be included in a heterogeneous mixing of infection model. Consider two subpopulations of sizes N_1 and N_2 respectively, each divided into susceptibles and infected members with subscripts to identify the subpopulation. Suppose that group i members make a_i contacts in unit time and that the fraction of contacts made by a member of group i that is with a member of group j is p_{ij} ,i, j = 1, 2. For the properties of the mixing matrix, $p_{11} + p_{12} = p_{21} + p_{22} = 1$. A two-group model may describe a population with groups differing by activity levels and possibly by vulnerability to infection, so that $a_1 = a_2$ but $\gamma_1 \neq \gamma_2$. It may also describe a population with one group which has been vaccinated against infection, so that the two groups have the same activity level but different disease model parameters. In this case, $a_1 = a_2$ but $\gamma_1 = \gamma_2$.

An infection model with two subgroups is given as,

$$\begin{aligned} S_1'(t) &= -a_1 \tfrac{S_1}{N_1}\Big(p_{11}I_1 + p_{12}I_2\Big) \\ I_1'(t) &= a_1 \tfrac{S_1}{N_1}\Big(p_{11}I_1 + p_{12}I_2\Big) - \gamma_1 I_1 \\ S_2'(t) &= -a_2 \tfrac{S_2}{N_2}\Big(p_{21}I_1 + p_{22}I_2\Big) \\ I_2'(t) &= a_2 \tfrac{S_2}{N_2}\Big(p_{21}I_1 + p_{22}I_2\Big) - \gamma_2 I_2 \end{aligned} \tag{1}$$

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