Positive solutions for elliptic equations in two dimensions arising in a theory of thermal explosion

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Abstract

We study a mathematical model of thermal explosion which is described by the boundary value problem

$$\begin{cases} -\Delta u = \lambda e^{u^\alpha}, & x \in \Omega, \\ \mathbf{n} \cdot \nabla u + g(u)u = 0, & x \in \partial\Omega, \end{cases}$$

where the constant $\alpha \in (0, 2]$, $g : [0, \infty) \to (0, \infty)$ is an increasing C^1 function, Ω is a bounded domain in $\mathbb{R}^2$ with smooth boundary $\partial\Omega$ and $\lambda > 0$ is a bifurcation parameter. Using variational methods we show that there exists $0 < \Lambda < \infty$ such that the problem has at least two solutions if $0 < \lambda < \Lambda$, no solution if $\lambda > \Lambda$ and at least one solution when $\lambda = \Lambda$.

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