Axial Green's function Method(AGM) and Localized AGM for incompressible flow

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ABSTRACT

When calculating the incompressible flows in complicated domains, we have to carefully discretize the equations of fluid motion in the domain in order to obtain the accurate numerical solutions of flow, the velocity and pressure. Inspired by the axial lines in the domain, we have developed the Axial Green's function Method(AGM) [2] to effectively implement the divergence-free condition for incompressible flows.

This method begins at first to effectively solve the general elliptic boundary value problems in arbitrary domains [1]. It has an outstanding advantage that one-dimensional Green's functions for the axially decomposed differential operators from the flow equations yield an innovative formulation for the solution of incompressible flow. This method maintains the second-order convergence of the fluid velocity.

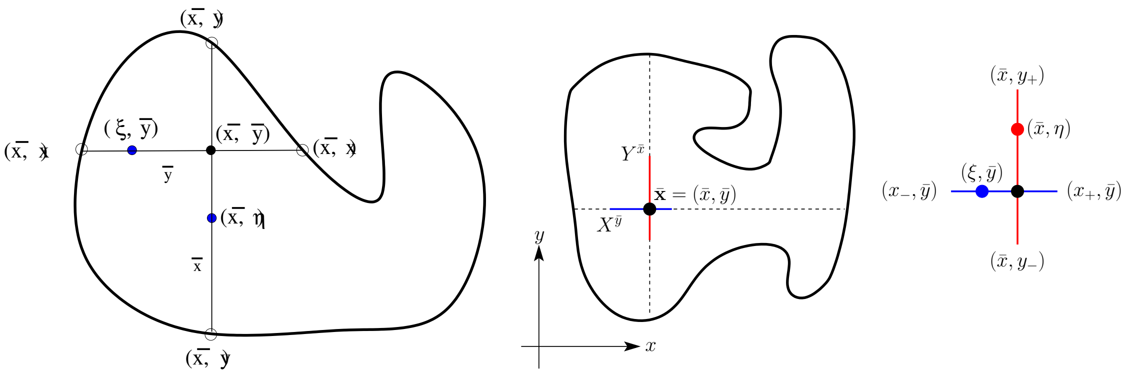
However, the AGM produces a quasi-sparse matrix which is caused by the maximal axial lines. Enforcing boundary conditions becomes another issue in this case. From these reasons, the localized AGM has been developed recently for the convection-diffusion equations [3] which could be a preliminary work on the way of the incompressible Navier-Stokes flow.

This talk provides comprehensive features of the AGM and the localized AGM from the viewpoint of a novel numerical method. Particularly, they are shown as a potential method for incompressible flows.

AGM TO LOCALIZED AGM

The AGM and the localized AGM are simply characterized by the axial lines for computations. As shown in Fig. 1, if we use the maximal axial lines in (a), then the method is said to be the AGM. Otherwise, i.e., if the local axial lines in (b) are employed, then we call the method the localized AGM.

These methods use one-dimensional Green's functions on axial lines to solve higher dimensional(2D/3D) problems of interest in arbitrary domains, which are usually modelled by partial differential equations. Therefore, only one-dimensional integrations are needed over all computations. This feature becomes a good advantage of these methods. The axial lines are much easier to construct compared to grids in FDM or meshes in FEM.



(a) Maximal axial lines for AGM (b) Local axial lines for AGM

Figure 1. Axial lines

INCOMPRESSIBLE FLOWS

The velocity $\mathbf{u} = (u, v)$ of an incompressible flow has to satisfy the divergence-free condition as follows:

$$u_x + v_y = 0.$$

It is well known that the pressure of an incompressible flow like the Stokes flow or the Navier-Stokes one is closely related to the divergence-free. In many numerical methods, the pressure correction strategy is the key in calculating the flow.

In this talk, we propose an innovative formulation for the pressure correction using not only the AGM but also the localized AGM.

REFERENCES

1. D. W. Kim, S.-K. Park, and S. Jun, "Axial Green's function method for multi-dimensional elliptic boundary value problems", *International Journal for Numerical Methods in Engineering*, Vol. 76, 2008, pp. 697-726.
2. S. Jun and D.W. Kim, "Axial Green's Function Method for Steady Stokes Flow in Geometrically Complex Domains", *Journal of Computational Physics*, Vol. 230, 2011, pp. 2095-2124.
3. W. Lee and D.W. Kim, "Localized Axial Green's Function Method for the Convection-Diffusion Equations in Arbitrary Domains", *Journal of Computational Physics*, Vol. 275, 2014, pp. 390-414.