Strong monogamy of multi-party quantum entanglement

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ABSTRACT

We provide a strong evidence for strong monogamy inequality of multi-qubit entanglement recently proposed in [B. Regula *et al.*, Phys. Rev. Lett. **113**, 110501 (2014)]. We consider a large class of multi-qubit generalized W-class states, and analytically show that the strong monogamy inequality of multi-qubit entanglement is saturated by this class of states.

PRIMARY HEADING

Whereas classical correlation can be freely shared among parties in multi-party systems, quantum entanglement is restricted in its shareability; if a pair of parties are maximally entangled in multipartite systems, they cannot have any entanglement nor classical correlations with the rest of the system. This restriction of entanglement shareability among multi-party systems is known as the *monogamy of entanglement* (MoE) [1].

The first mathematical characterization of MoE was established by Coffman-Kundu-Wootters (CKW) for three-qubit systems [2] as an inequality; for a three-qubit pure state $|\psi\rangle_{ABC}$ with its one-qubit and two-qubit reduced density matrices $\rho_A = \mathrm{tr}_{BC}|\psi\rangle_{ABC}\langle\psi|$, $\mathrm{tr}_C|\psi\rangle_{ABC}\langle\psi| = \rho_{AB}$ and $\mathrm{tr}_B|\psi\rangle_{ABC}\langle\psi| = \rho_{AC}$ respectively,

$$\tau\left(|\psi\rangle_{A|BC}\right) \geq \tau\left(\rho_{A|B}\right) + \tau\left(\rho_{A|C}\right), \tag{1}$$

where $\tau\left(|\psi\rangle_{A|BC}\right)$ is the *tangle* of the pure state $|\psi\rangle_{ABC}$ quantifying the bipartite entanglement between A and BC, and $\tau\left(\rho_{A|B}\right)$ and $\tau\left(\rho_{A|C}\right)$ are the tangles of the two-qubit reduced states $\rho_{AB} = \mathrm{tr}_C|\psi\rangle_{ABC}\langle\psi|$ and $\rho_{AC} = \mathrm{tr}_B|\psi\rangle_{ABC}\langle\psi|$, respectively. Later, CKW inequality was generalized for multi-qubit systems [3] and some cases of higher-dimensional quantum systems in terms of various entanglement measures [4][5][6][7].

Recently, a *strong monogamy*(SM) inequality of multi-qubit entanglement was proposed by conjecturing the nonnegativity of the n-tangle [8]. For the validity of SM inequality, an extensive numerical evidence was presented for four qubit systems together with analytical proof for some cases of multi-qubit systems. However, proving SM conjecture analytically for arbitrary multi-qubit states seems to be a formidable challenge due to the numerous optimization processes arising in the definition of n-tangle. Here we consider a large class of multi-qubit states, *generalized W-class states*, and analytically show that SM inequality proposed in [8] is saturated by this class of states.

STRONG MONOGAMY INEQUALITY

Strong monogamy of multi-qubit entanglement was proposed as [8],

$$\tau\left(|\psi\rangle_{A_1|A_2\cdots A_n}\right) \geq \sum_{m=2}^{n-1}\sum_{\vec{j}^m}\tau\left(\rho_{A_1|A_{j_1^m}|\cdots|A_{j_{m-1}^m}}\right)^{m/2}, \tag{2}$$

where the index vector $\vec{j}^m = (j_1^m, \ldots, j_{m-1}^m)$ spans all the ordered subsets of the index set $\{2, \ldots, n\}$ with $(m-1)$ distinct elements, and for each $m = 2, \cdots, n-1$, the m-tangle for multi-qubit mixed state is defined by convex-roof extension,

$$\tau\left(\rho_{A_1|A_{j_1^m}|\cdots|A_{j_{m-1}^m}}\right) = \left[\min_{\{p_h, |\psi_h\rangle\}}\sum_h p_h\sqrt{\tau\left(|\psi_h\rangle_{A_1|A_{j_1^m}|\cdots|A_{j_{m-1}^m}}\right)}\right]^2, \tag{3}$$

with the minimization over all possible pure state decompositions

$$\rho_{A_1A_{j_1^m}\cdots A_{j_{m-1}^m}} = \sum_h p_h|\psi_h\rangle_{A_1A_{j_1^m}\cdots A_{j_{m-1}^m}}\langle\psi_h|. \tag{4}$$

Here we show that the inequality (6) is saturated by multi-qubit W-class states [9]

$$|\psi\rangle_{A_1A_2\ldots A_n} = a|00\cdots0\rangle + b_1|10\cdots0\rangle + b_2|01\cdots0\rangle + \ldots + b_n|00\cdots1\rangle. \tag{5}$$

In other words, for any W-class state in Eq. (5), we have

$$\tau\left(|\psi\rangle_{A_1|A_2\cdots A_n}\right) = \sum_{m=2}^{n-1}\sum_{\vec{j}^m}\tau\left(\rho_{A_1|A_{j_1^m}|\cdots|A_{j_{m-1}^m}}\right)^{m/2}. \tag{6}$$

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