ON STABILITY AND GENERICITY PROPERTIES FOR POLYNOMIAL OPTIMIZATION PROBLEMS

Gue Myung LEE [1] and Pham Tien SON [2]

1) *Department of Applied Mathematics, Pukyong National University, Busan 608-737, KOREA*

2) *Department of Mathematics, University of Dalat, 1 Phu Dong Thien Vuong, Dalat, VIETNAM*

Corresponding Author : Gue Myung LEE, gmlee@pknu.ac.kr

ABSTRACT

Recently, various continuity and differentiability properties of the global solution map, the local solution map, the Karush-Kuhn-Tucker set-valued map, and the optimal value function of the quadratic programs have been established (see [1–6], and the references therein). Since the quadratic optimization problems can be regarded as very special forms of polynomial optimization problems, we may ask questions about stability properties for global solution maps and of Karush-Kuhn-Tucker set-valued maps and optimal value functions for perturbed polynomial optimization problems. Moreover, genericity properties for optimization problems have been studied (see [7,8]).

In this talk, using some basic theorems in semi-algebraic geometry ([9,10]), for example, Tarski-Seidenberg Theorem and Sard's Theorem with parameter, etc, we establish the strong Hölder stability with exponent explicitly determined, the upper semicontinuity and the lower semicontinuity of global solution map $Sol(\cdot)$ and of Karush-Kuhn-Tucker set-valued map $KKT(\cdot)$ for polynomial optimization problems with perturbed objective functions. We also give explicit formulas for computing the directional derivative and the subgradient, the (Fréchet) derivative of the optimal value function $\phi(\cdot)$ for polynomial optimization problems with perturbed objective functions. Moreover, we present the generic continuity of the set-valued maps $Sol(\cdot)$ and $KKT(\cdot)$, and the generic differentiability of the function $\phi(\cdot)$.

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