

A BDDC algorithm with optimally enriched primal unknowns

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ABSTRACT

It is well known that the standard BDDC algorithm [2] requires a strong assumption on coefficients of the model problem related to the subdomain partition to achieve a good performance. In the works by Clemense and Scheichl [3,4], a bound of condition number of FETI algorithms has been analyzed depending on the coefficient variations inside subdomains. A similar problem was also considered in the work [5] and an additional set of primal unknowns are introduced as in [6–8]. In this paper, to address a fast and robust solver for second order elliptic problems with highly varying coefficients, an adaptive BDDC algorithm is developed and analyzed. To obtain a robust algorithm based on the classical BDDC approach, some enriched primal unknowns are constructed. The primal unknowns consist of a standard component and an enriched component. The standard component is the set of basis functions corresponding to subdomain vertices, which is the building block of the classical BDDC algorithm. The enriched component of the primal unknowns are chosen adaptively to reflect the local structures of the coefficient by solving two types of generalized eigenvalue problems. These problems are solved locally so that, for every two adjacent subdomains, two types of generalized eigenvalue problems are defined. Given a tolerance, dominant eigenfunctions with eigenvalues larger than this tolerance are pre-computed and taken as primal unknowns. We prove that the resulting condition number by using this enriched coarse space is bounded above by this tolerance, independent of the contrast of the given coefficient. Numerical results are presented for various model problems and various choices of primal unknowns to show the performance of the proposed preconditioner. In addition, we discuss about a possible improvement on our method by using parallel sum of matrices [1] and deluxe scaling.

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