SUPERCONVERGENCE AND A POSTERIORI ERROR ESTIMATION

Kwang-Yeon Kim [1]

1) *Department of Mathematics, Kangwon National University, Chuncheon, Korea*

ABSTRACT

Superconvergence has been one of the active research topics in the finite element community because it yields a cheap way of improving the accuracy of the computed finite element approximation u_h. The most common form of superconvergence is the supercloseness between (1) u_h and some interpolation of the exact solution u or (2) u_h and u at special points of each element. In this talk we introduce the well-known superconvergence result for the linear triangular finite element method of second order elliptic equations and discuss how it is utilized in a posteriori error estimation. A posteriori error estimation plays a central role in adaptive mesh refinement by providing a computable quantity η which estimates the error $|||u - u_h|||$ (with respect to some norm $|||\cdot|||$) in terms of the computed finite element solution u_h and the known data. The best result we can obtain is that η is asymptotically exact, namely, $\eta/|||u - u_h||| \to 1$ as the mesh size h goes to 0. We investigate some popular error estimators with this property both theoretically and numerically.

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