# Nonlinear Beam/Shell Analysis for Flapping Wing via Co-rotational Framework

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## ABSTRACT

This paper presents a non-linear beam and shell analyses for a flapping wing prediction using the co-rotational (CR) framework. The CR framework allows for geometrically non-linear structural analysis based on the assumptions on small strain and large displacement. Using the unified analytical CR framework, non-linear CR beam and shell elements are established. It is validated by comparing with the results from commercial software in static condition and transient analysis, in part.

## INTRODUCTION

The flapping wings accommodate a large angular motion with significant nonlinear deformation. In order to analyze those behaviors, precisely, a sophisticated non-linear structural analysis is required. The CR framework, based on the assumptions on small strain and large displacement, allows for geometrically non-linear structural analysis [1]. Also, it is capable to consider a large angular motion by using the three-dimensional rotational parameter. Due to those features, there are some previous researches regarding the analysis of flapping wing by using the CR framework [2]. However, their analytical modeling of the wing is insufficient to make a tracing of the realistic geometry, e.g., vein upon the wing, which has complex cross section. Therefore, a realistic structural modeling capability with precision analysis is still required. Currently, the three-dimensional beam element with warping degree of freedom operated in both static and dynamic analyses is developed. The three-node CR shell element in the static analysis is developed as well. Regarding those analyses, a unified analytical CR framework is used. The present analyses will be validated by comparing with the results from commercial software. The present analyses will be refined to analyze the realistic configuration of flapping wing.

## FORMULATION AND NUMERICAL RESULTS

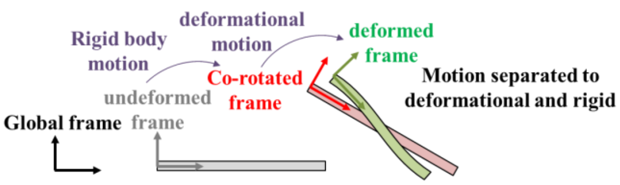


Figure 1. Coordinates used in the present CR framework

Figure 1 shows the coordinate upon the framework. The CR framework decouples the geometrical non-linearity from the material response of the element. A basic idea of this approach is to extract pure elastic deformation from the total motion using a co-rotational frame (CR frame) which translates and rotates along the element. A consistent transformation of element stiffness matrix and internal force vector from the local CR frame to the global coordinate system is used, while the geometrically nonlinear effects are considered. Details of unified CR framework's mathematics, presented in Ref. [1], can be summarized as follows:

$$f_g = EP^T f_l, \qquad K_g = E[P^T K_l P - GF_1^T P - F_2 G^T]E^T \tag{1}$$

where, $f$ and $K$ mean internal force vectors and stiffness matrix, respectively. $E$ and $P$ represent rotation matrix, transforming between the local and global frame, and projection matrix, extracting deformational part from the total motion. By following the above manner as well as the consideration of nodal DOF, it is possible to compose various nonlinear finite elements. In this paper, the Euler Bernoulli and Timoshenko hypotheses are employed, respectively, in order for the linear element of CR beam analyses. By considering the warping DOF, 7 by 7 stiffness matrix is constructed for each node. Also, OPT-DKMT shell [3] is employed in order for the linear element of CR shell analysis. The classical nonlinear analysis example, cantilevered plate with the various tip moments, is examined by using both CR beam and shell analyses. The comparison of the results from two analyses is illustrated in Fig. 2. The present results show good correlation with that from the analytical solution. Figure 3 shows a comparison of transient response of the plate with harmonic plunge motion. The present result of CR beam analysis is compared with that analyzed using solid element by ANSYS. The present result shows good correlation with the peak to peak difference as 0.14% in z-direction.

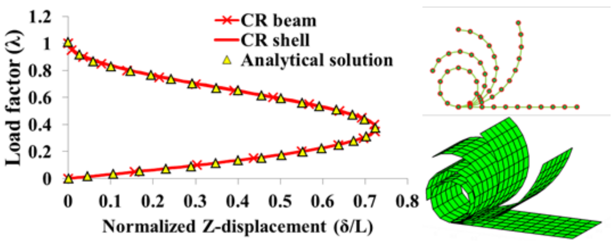


Figure 2. Comparison on the vertical deflection and deformed shape

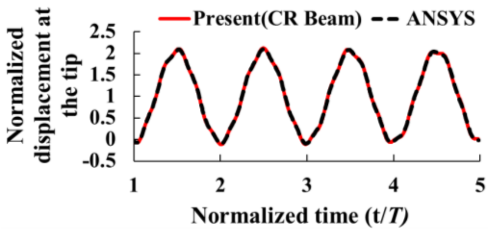


Figure 3. Comparison on the response of vertical tip deflection

The present CR shell will be expanded to be operated in time-domain. Finally, the CR shell and beam elements will be applied to analyze the realistic flapping wing.

## REFERENCES


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