CONVERGENCE OF RELAXED METHOD FOR SOLVING NONLINEAR MATRIX EQUATION

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ABSTRACT

We consider a matrix polynomial equation which has the form of

$$A_nX^n + A_{n-1}X^{n-1} + \cdots + A_0 = 0$$

where A_n, A_{n-1}, ... , A_0 and X are square matrices. Matrix polynomials occur in the theory of differential equations, system theory, network theory, stochastic theory and other areas. In a quadratic case ($n = 2$), Higham and Kim [1, 2] incorporated the exact line searches into Newtons method to improve the global convergence properties. Also Latouche [3] considered matrix polynomials with infinite degree (i.e. $n = \infty$) (so-called power series matrix equation) which arise in Markov chain and he showed that Newton's method for solving the matrix polynomial equation converges to the elementwise minimal nonnegative solution. The definition of elementwise minimal positive and nonnegative solutions are following:

Definition 1. Let F be a matrix function from $\mathbb{R}^{m\times n}$ to $\mathbb{R}^{m\times n}$. Then a positive (-nonnegative) solution S_1 of the matrix equation $F(X) = 0$ is an elementwise minimal positive (-nonnegative) solution and a positive (-nonnegative) solution S_2 of $F(X) = 0$ is an elementwise maximal positive (-nonnegative) solution if for any positive (-nonnegative) solution S of $F(X) = 0$,

$$S_1 \leq S \leq S_2$$

where for any matrices $A = [a_{ij}], B = [b_{ij}] \in \mathbb{R}^{m\times m}$, we write $A \geq B$ ($A > B$) if $a_{ij} \geq b_{ij}$ ($a_{ij} > b_{ij}$) holds for all i, j.

In this paper the monotone convergence of Newton's method for solving the equations is considered and we show that the elementwise minimal nonnegative solution and the symmetric positive definite solution can be found by the methods, respectively. Moreover, the relaxed method preserving the monotonicity result is provided. Finally, numerical experiments show that our method reduces the number of iterations significantly.

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