ON THE CONVERGENCE RATE ANALYSIS OF CYCLIC COORDINATE GRADIENT DESCENT METHODS

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ABSTRACT

A Cyclic coordinate gradient descent (CCGD) method is simple, stable, and fast at each iteration. This method has also shown its competitive performance on separable nonsmooth convex minimization problems, whose objective function is the sum of a smooth function and a separable (and possibly nonsmooth) convex function, such as the ℓ_1-regularized linear least squares problem and the ℓ_1-regularized logistic regression problem. Hence the CCGD method has obtained much attention in applications of applied mathematics, statistics and engineering. But, its global convergence rate has not fully been analyzed. We prove that the sublinear convergence rate for the CCGD method is $O(1/k)$, where k is the iteration counter, when the smooth function of the objective has a Lipschitz gradient. Also, the linear rate convergence of the method is proved when the objective is a strongly convex function having a Lipschitz gradient or when the smooth function of the objective is a composition of a strong convex function having a Lipschitz gradient with a linear function, the convex function of the objective is polyhedral, and there is a real number whose corresponding level set of the convex function contains the set of optimal solutions and is bounded.

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